Price Level and Inflation Dynamics in Heterogeneous Agent Economies

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Abstract

We study price level dynamics in a heterogeneous-agent incomplete-market economy with nominal government debt and flexible prices. Unlike in representative agent economies, steady-state equilibria exist when the government runs permanent deficits, provided the level of deficits is not too large. We quantify the maximum sustainable deficit for the US and show that it is lower under more redistributive tax and transfer systems. With constant primary deficits, there exist two steady-states, and the price level and inflation are not uniquely determined. We describe alternative policy settings that deliver uniqueness. We conduct quantitative experiments to illustrate how redistribution and precautionary saving amplify price level increases in response to deficit expansions and fiscal helicopter drops. We show that rising primary deficits can account for a decline in the long-run real interest rate, leading to permanently higher inflation. Our work highlights the role of household heterogeneity and market incompleteness in determining inflation and interest rate dynamics.

Keywords: Fiscal theory of the price level, Heterogeneity, Incomplete markets, Inflation, Precautionary saving, Redistribution, Sustainable deficit.

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1 Introduction

We develop a framework to study the causes and consequences of price level dynamics in an economy with three features: (i) a fiscal authority issues nominal debt to finance real expenditures and transfers to households; (ii) a monetary authority sets the short-term nominal rate on government debt; (iii) financial markets are incomplete, so households have a precautionary motive to accumulate savings in order to self-insure against idiosyncratic income risk. We focus on a class of fiscal rules which fall under the framework of the *Fiscal Theory of the Price Level* (FTPL) and study conditions under which they give rise to a determinate price level and inflation.[1]

This approach has proven useful for analyzing the recent inflation bout which followed large debt-financed fiscal expansions—aimed at cushioning the COVID-19 supply shock—and sharp interest-rate movements by central banks around the world. These fiscal policy rules, however, have been applied almost exclusively to representative agent economies. Our motivation for extending this analysis to "Bewley" economies (Bewley, 1986) with heterogeneous agents and incomplete markets is threefold. First, heterogeneous agent models generate consumption responses to income and interest rates that are consistent with the vast body of micro-economic evidence on the joint dynamics of household income and spending.[2] This property is important because household spending pressure is a key force shaping inflation and interest rates in equilibrium.

Second, household heterogeneity has played an important role in both the drivers and consequences of the latest inflationary episode. Governments issued vast quantities of new debt to finance transfers that were targeted to certain groups of households. The ongoing spending pressures that are leading many governments to run persistent deficits are also highly targeted. Heterogeneous agent models are a natural environment to study the implications of such interventions, as well as the distributional effects of shocks and subsequent policy responses.

[1] The FTPL literature, which has its roots in Sargent and Wallace (1981) and builds on Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (1998) is too vast to cite in full. See the handbook chapter by Leeper and Leith (2016) and book by Cochrane (2023) for a synthesis of the reach of FTPL models.

[2] See for example the review article by Kaplan and Violante (2022).

Third, working in a heterogeneous-agent incomplete-market setting also overcomes a limitation of representative agent FTPL models that makes their application to current macroeconomic conditions problematic. Standard representative agent models require governments to run positive primary surpluses in expectation at all points in time. However, in recent decades the US and many other developed economies have consistently run primary deficits, and their fiscal positions look unlikely to return to surpluses anytime soon. Heterogeneous agent versions of these models offer a natural setting in which to study price level dynamics with persistent primary deficits. In these versions, the real return on government debt r can be less than the growth rate of the economy g, which is also a feature of recent macroeconomic conditions.

This motivation leads us to start building a bridge between the well-studied representative-agent FTPL and workhorse heterogeneous-agent models in the tradition of Bewley (1986), Imrohoroğlu (1989), Huggett (1993) and Aiyagari (1994). In this paper, we take a first step by focusing on flexible-price economies.

Theoretical Analysis. We first analyze an endowment economy in which the government runs positive primary surpluses and $r > g$. Here, the conditions on fiscal and monetary policy for the price level and inflation to be uniquely determined are essentially unchanged from corresponding representative agent economies. There are, however, important quantitative differences that reflect the role of precautionary savings. Unlike in the representative agent economy, in the heterogeneous agent economy changes in fiscal policy lead to movements in the real interest rate. This is because a change in either the level of debt, or the size and distribution of surpluses alters the overall demand for savings among households. For a given setting of monetary policy, these different real rate dynamics imply different paths of inflation. It also means that there are non-trivial inflation dynamics following a one-time fiscal helicopter drop, and that the path of inflation depends on the targeting of the fiscal injection.

We then analyze the same heterogeneous-agent economy but with a government that runs a constant primary deficit and $r < g$. We show that, as long as the level of deficits is not too large, equilibria with a finite price level where debt is valued exist. The maximum possible level of deficits is decreasing in the amount of redistribution implicit in the tax and transfer system. For example, higher lump-sum transfers—which reduce the volatility of households' disposable income—lower aggregate precautionary saving and thus the demand for government debt as a savings instrument. Instead, expanding deficits by cutting proportional taxes may have the opposite effect because it increases the volatility of disposable income. Thus, the maximum sustainable deficit is higher in this latter case.

For levels of deficits below this maximum level, there are generally two steady-states. Therefore, without additional assumptions, standard FTPL arguments do not uniquely pin down the price level or the path of inflation. Numerically, we have found the low inflation steady-state is saddle-path stable: there is a unique initial price level and subsequent path of inflation and real rates leading to that steady-state. In contrast, the high inflation steady-state is locally stable: there is a continuum of initial price levels that support paths of inflation leading to that steady-state.

To illustrate the key economics of our framework, we also benchmark our findings to a simple bonds-in-utility (BIU) economy. In this economy, households derive a "convenience yield" from holding bonds, which is meant to capture the inelastic steady-state household asset demand in our heterogeneous agent economy in a reduced form way (Angeletos et al., 2023a)—as interest rates increase, so too does the total quantity of debt that households demand. We employ our BIU model to transparently highlight the economic forces behind determinacy and inflation dynamics in our heterogeneous agent framework. Nonetheless, the BIU setting is not well suited to studying how redistribution and precautionary motives shape inflation dynamics following fiscal policy changes. We therefore rely on a calibrated version of our Bewley economy to quantify the importance of these forces.

Quantitative Policy Messages. First, we consider the effects of permanently increasing deficits. We calculate that if the government were to permanently increase lump sum transfers to households without ever raising taxes, the largest sustainable deficit would be approximately 4.2% of GDP, or 120% higher than the 2014-2019 average. The maximum sustainable deficit depends on the degree of social insurance: expanding deficits in a more progressive manner implies lower maximum deficits. The reason is that tax systems that provide more social insurance weaken precautionary savings, thus lowering household demand for government debt. More progressive tax systems therefore reduce fiscal space.

A permanently higher deficit is associated with a lower steady-state real interest rate and less real government debt, as well as a higher long-run inflation rate for a given nominal rate target. This is because a larger deficit must be funded by larger real interest receipts, which require a more negative real rate. Quantitatively this discrepancy with the representative agent model is large: a permanent deficit expansion from 2.0% to 2.5% of GDP lowers the long-run real rate between 0.7 and 0.8 percentage points. The heterogeneous agent framework thus offers an alternative interpretation of "secular stagnation" by linking together growing primary deficits, falling real rates, and rising inflation.

We illustrate that an increase in deficits financed by lump-sum transfers depresses the long-run real rate by more than if the financing occurs via a tax cut. Even small differences in the long-run real rate due to the progressivity of financing can compound to cause substantial changes in the initial jump in the price level: the price level jumps by approximately 30% more when the government raises transfers rather than cut taxes. We also emphasize that, to keep long-run inflation unchanged, permanent fiscal expansions necessitate a corresponding decline in the monetary authority's long-run nominal interest rate target.

Next, we study the effects of issuing new debt while holding primary deficits constant: a fiscal helicopter drop of around 14% of annual GDP, roughly the size of the fiscal expansion in the US over the course of the COVID-19 pandemic. Consistent with the representative agent experiments in Cochrane (2022), we find that this generates an immediate jump in the price level. However, relative to the representative agent benchmark, in our economy the initial increase in the price level is 30% larger. This amplification is driven by redistribution and heterogeneity of marginal propensities to consume (MPC) as the dilution of nominal debt entails large amounts of redistribution from wealthy to poor households. This wealth reallocation puts upward pressure on consumption, which increases real rates and interest payments on government debt, thereby causing a larger initial jump in the price level. A targeted helicopter drop such as that implemented in the US, which targets high MPC households, fuels additional short-term inflationary pressures.

Related Literature. Our paper belongs to a small but growing literature that moves beyond the representative agent model and explores the FTPL with incomplete markets. Bassetto and Cui (2018) show that a model of overlapping generations and a model in which government debt provides special liquidity services can give rise to multiple steady-states in which the real interest rate on government debt is below the growth rate of output. They emphasize that the FTPL can fail to yield price level determinacy in these settings. Angeletos et al. (2023b) and Angeletos et al. (2024) study a wide class of fiscal policies in an overlapping generations model with nominal rigidities, while Miao et al. (2025) study the impact of fiscal deficits in an overlapping generations economy when $r < g$. Brunnermeier et al. (2020, 2024), Miao and Su (2021) and Amol and Luttmer (2022) all study models with idiosyncratic risk in the rate of return on capital, and explore settings for fiscal policy that can establish price level uniqueness in low interest rate environments.

Our work differs from these papers in three respects. First, we investigate the implications of the FTPL in a Bewley (1986) economy in which market incompleteness arises from uninsurable labor income risk. Second, we show how fiscal policy can still give rise to price level uniqueness in models where the government runs persistent primary deficits. Third, we quantitatively explore the response of economic aggregates to unanticipated shocks in low-interest rate economies with persistent deficits. To the best of our knowledge, the messages we deliver about the key role of precautionary savings and MPC heterogeneity in driving price level, inflation and real rate dynamics in this class of economies are novel.[3]

Closer to our analysis is Hagedorn (2021), who also explores price level determination in a Bewley economy with nominal government debt, but focuses on a different class of fiscal policies outside of the FTPL. Relative to Hagedorn (2021), we also explore how precautionary savings and MPC heterogeneity drive price level dynamics, as well as how these features shape the government's maximum permanent deficit.

Our paper also relates to the literature that studies the implications of low interest

[3]Some qualitative aspects of our analysis, such as equilibrium multiplicity with deficits, share features with certain monetarist economies. See, for example, Chapter 18 of Ljungqvist and Sargent (2018).

rate environments for government borrowing (Blanchard, 1985, 2019; Cochrane, 2021; Aguiar et al., 2021; Mehrotra and Sergeyev, 2021; Mian et al., 2021a; Reis, 2021; Kocherlakota, 2023; Miao et al., 2025). This body of work emphasizes that the government can roll over debt indefinitely when the real interest rate on government debt is below the growth rate of the economy, provided that the amount of debt is not too large. We quantify this bound in our calibrated model for the U.S. economy and illustrate how it depends on the level of uninsurable income risk and on the degree of fiscal redistribution.

The insight that the size of fiscal space depends on the level of idiosyncratic risk and inequality is shared with Mian et al. (2021a), Reis (2021), Amol and Luttmer (2022), and Brunnermeier et al. (2020, 2024). Relative to these papers, we emphasize that *how* the government finances deficits can change the strength of precautionary savings and thus the government's maximum implementable deficit. In particular, the presence of uninsurable income risk allows us to explore this issue when the government can raise revenues through both lump-sum and proportional taxation. In complementary work, Bayas-Erazo (2023), studies how the interaction between income risk and redistributive taxation affects optimal fiscal policy when the government maximizes a social welfare function.[4]

Finally, our paper highlights that unanticipated changes in the price level can give rise to persistent dynamics in the real interest rate and inflation due to heterogeneous wealth effects across the distribution. As such, it builds on work that explores the distributional consequences of surprise inflation (Doepke and Schneider, 2006). Relatedly, Bilbiie et al. (2013) and Hagedorn et al. (2019), *inter alia*, emphasize that redistribution and the targeting of fiscal transfers can matter for aggregate output in models with nominal rigidities. We show that distributional effects of fiscal policy has important consequences for inflation dynamics even in a flexible price environment.

[4] Angeletos et al. (2023a) also study optimal fiscal policy in environments in which debt provides liquidity provisions, as in the bonds-in-utility economy we consider in Section 3.

2 Model Environment

2.1 Households

Demographics. Time is continuous and is indexed by $t \geq 0$. The economy is populated by a continuum of households indexed by $j \in [0, 1]$.

Endowments. Real aggregate output y_t is exogenous and grows at a constant rate $g \geq 0$. Household j receives a stochastic share $z_{jt} > 0$ of aggregate output. The shares z_{jt} are independent across households and a law of large numbers holds so that there is no economy-wide uncertainty,

$$\int_{j \in [0,1]} z_{jt} \mathrm{d}j = 1 \text{ for all } t \geq 0. \tag{1}$$

In our baseline model we assume that z_{jt} follows an N-state Poisson process taking values in $z \in \mathcal{Z} \subset \mathbb{R}_{++}$, with switching intensities $[\lambda_{z,z'}]_{z,z' \in \mathcal{Z}}$.

Assets. Households trade a short-term risk-free bond that yields a nominal flow return i_t. We denote the nominal bond holdings of household j at time t by A_{jt}. This asset is the unit of account in the economy, and we let P_t denote the price of output in terms of this short-term bond.

Preferences. Households take the path of aggregate variables $\{P_t, i_t, y_t\}_{t \geq 0}$ as given and choose real consumption flows $\tilde{c}_{jt}$ to maximize

$$\mathbb{E}_0 \int_0^\infty e^{-\tilde{\rho} t} \frac{\tilde{c}_{jt}^{1-\gamma}}{1-\gamma} \mathrm{d}t \tag{2}$$

with $\gamma \geq 0$, where the expectation is taken over the idiosyncratic endowment process z_{jt}. We denote the household's discount rate by $\tilde{\rho} > 0$.

Nominal Household Budget Constraint. The initial distribution of nominal assets A_{j0} is given. For $t > 0$, households face a flow budget constraint

$$\mathrm{d}A_{jt} = \left[i_t A_{jt} + (z_{jt} - \tau_t(z_{jt}))\, P_t y_t - P_t \tilde{c}_{jt} \right] \mathrm{d}t. \tag{3}$$

where the path of the tax and transfer (or net tax) function, set by the fiscal authority, satisfies $\tau(z) < z$.[5] Nominal savings $\mathrm{d}A_{jt}$ are equal to the sum of asset income $i_t A_{jt}$ and endowment income net of taxes and transfers $(z_{jt} - \tau_t(z_{jt}))\, P_t y_t$, minus consumption expenditures $P_t \tilde{c}_{jt}$. In our baseline model we assume that households cannot borrow, $A_{jt} \geq 0$, but we relax this assumption in Section 5.

Price Level and Inflation. There are no price adjustment frictions. In this flexible-price economy, the price level P_t may exhibit jumps. We consider perfect-foresight solutions in which the price level follows a continuously differentiable path for $t > 0$ and define the inflation rate π_t as:

$$\frac{\mathrm{d}P_t}{P_t} = \pi_t \mathrm{d}t \tag{4}$$

This does not impose any restrictions on whether or not the initial value of the price level P_0 is determinate. The price level may jump in response to unanticipated shocks.

De-trended Real Household Budget Constraint. We denote de-trended real assets and de-trended real consumption as

$$a_{jt} := \frac{A_{jt}}{P_t y_0 e^{gt}} \qquad c_{jt} := \frac{\tilde{c}_{jt}}{y_0 e^{gt}} \tag{5}$$

For $t > 0$, we can re-write the nominal budget constraint (3) in de-trended real terms:

$$\mathrm{d}a_{jt} = \left[r_t a_{jt} + z_{jt} - \tau_t(z_{jt}) - c_{jt}\right] \mathrm{d}t \tag{6}$$

where $r_t := i_t - \pi_t - g$ is the growth-adjusted real rate. At $t = 0$, de-trended real assets a_{j0} are given by the ratio of A_{j0} to the endogenous initial price level P_0.

Relative Asset Holdings. Let A_t and a_t denote aggregate nominal and aggregate de-trended real household assets, respectively:

$$A_t := \int_{j \in [0,1]} A_{jt} \mathrm{d}j \qquad a_t := \int_{j \in [0,1]} a_{jt} \mathrm{d}j.$$

[5] We specify the tax function $\tau(z)$ as a function of the endowment shares z_{jt} rather than the level of income $z_{jt} y_t$ to maintain consistency with balanced growth even when $\tau(\cdot)$ is not linear.

We denote the share of assets held by household j at time t by $\omega_{jt} := \frac{A_{jt}}{A_t} = \frac{a_{jt}}{a_t}$, with

$$\int_{j\in[0,1]} \omega_{jt}\mathrm{d}j = 1 \text{ for all } t \geq 0. \tag{7}$$

Recursive Formulation of Household Problem. Given paths of real rates r_t and net taxes τ_t, the household problem can be expressed recursively via the Hamilton-Jacobi-Bellman Equation (HJB; see Achdou et al., 2022):

$$\begin{aligned}\rho v_t(a,z) - \partial_t v_t(a,z) &= \max_{c\geq 0} \frac{c^{1-\gamma}}{1-\gamma} + \partial_a v_t(a,z)\left[r_t a + z - \tau_t(z) - c\right] \\ &\quad + \sum_{z'\neq z} \lambda_{z,z'}\left[v_t(a,z') - v_t(a,z)\right],\end{aligned} \tag{8}$$

together with the boundary condition $\partial_a v_t(0,z) \geq (z-\tau_t(z))^{-\gamma}$ that ensures that the borrowing constraint $a \geq 0$ is satisfied. The growth-adjusted discount rate ρ in (8) is defined as $\rho = \tilde{\rho} - (1-\gamma)g$. The optimal consumption function $c_t(a,z)$ that solves the HJB is defined by

$$c_t(a,z) = \left[\partial_a v_t\left(a,z\right)\right]^{-\frac{1}{\gamma}}. \tag{9}$$

The associated savings function is denoted by

$$\varsigma_t(a,z) := r_t a + z - \tau_t(z) - c_t(a,z) \tag{10}$$

If a value function $v_t(a,z)$ solves the HJB (8) and satisfies the boundedness condition

$$\lim_{T\to\infty} \mathbb{E}_T\left[e^{-\rho T} v_T(a_{jT}, z_{jT})\right] = 0, \tag{11}$$

then the stochastic process for consumption defined by (9) solves the sequence version of the household problem (2).[6]

The distribution of households across real asset holdings and endowment shares

[6] See Theorem 3.5.3 in Pham (2009). The expectation in (11) is with respect to the stochastic process for idiosyncratic income and assets for household j, given by the budget constraint (6).

$g_t(a, z)$ satisfies the Kolmogorov Forward Equation (KFE)

$$\partial_t g_t(a, z) = -\partial_a \left[g_t(a, z) \varsigma_t(a, z) \right] - g_t(a, z) \sum_{z' \neq z} \lambda_{z,z'} + \sum_{z' \neq z} \lambda_{z',z} g_t(a, z'). \tag{12}$$

Let $f_t(\omega, z)$ denote the distribution of households across asset and endowment shares. For a given path of aggregate real wealth a_t, $f_t(\omega, z)$ and $g_t(a, z)$ are related by

$$f_t(\omega, z) = g_t(\omega a_t, z). \tag{13}$$

The KFE is a backward-looking equation where the initial distribution of endowment shares $f_0(\omega, z)$ is given from initial conditions, and the initial level of aggregate real wealth a_0 is determined as part of the equilibrium.

2.2 Government

Nominal Government Budget Constraint. We assume a fiscal authority that issues short-term nominal government debt B_t subject to the budget constraint:

$$\mathrm{d}B_t = \left[i_t B_t - (\mathcal{T}_t - x_t) P_t y_t \right] \mathrm{d}t \tag{14}$$

where $\mathcal{T}_t$ is the ratio of net tax revenues to output determined by

$$\mathcal{T}_t = \int_{j \in [0,1]} \tau_t \left(z_{jt} \right) \mathrm{d}j, \tag{15}$$

and $x_t \in [0, 1)$ is the proportion of total output that is spent as (wasteful) government expenditures. Observe that while $\tau_t(z)$ enters the household's budget constraint, x_t does not. We also define the ratio of primary surpluses to aggregate output as:

$$s_t = \mathcal{T}_t - x_t \tag{16}$$

Equation (14) defines the evolution of nominal government debt. This is a backward-looking equation where the initial level of nominal government $B_0 > 0$ is given. We restrict $B_t \geq 0$ so that the government can only borrow and not lend.

De-trended Real Government Budget Constraint. We denote de-trended real government debt (or the debt-output ratio) by b_t,

$$b_t = \frac{B_t}{P_t y_0 e^{gt}}. \tag{17}$$

For $t > 0$, real debt b_t evolves according to the real version of the government budget constraint given by (14):

$$\mathrm{d}b_t = [r_t b_t - s_t]\,\mathrm{d}t. \tag{18}$$

Real debt increases whenever real interest rate payments exceed real primary surpluses. At $t = 0$, de-trended real debt b_0 is a jump variable given by the ratio of exogenously given initial nominal debt B_0 to the endogenous initial price level P_0.

Fiscal Policy. For our baseline analysis we focus on a time-invariant net tax function τ^* and government expenditure share x^* so that surpluses are a constant fraction of real output, $s_t = s^*$ from (16). This is a natural starting point for our analysis because it delivers determinacy in representative agent economies with a positive surplus.[7] In Section 3.2, we allow for a broader class of fiscal rules of the form

$$s_t = s(b_t, r_t). \tag{19}$$

These rules allow primary surpluses to respond to real aggregate debt, real interest rates or real interest payments and play an important role in determining the price level when governments run persistent deficits, $s_t < 0$.

Monetary Policy. For our baseline analysis, we focus on a nominal interest rate peg $i_t = i^*$. As we explain below, different sequences for the nominal rate $\{i_t\}_{t\geq 0}$ give rise to different paths for inflation rates $\{\pi_t\}_{t\geq 0}$, but do not affect real outcomes.

2.3 Equilibrium

We first define a *real equilibrium* as a collection of real variables which satisfy household optimality, are consistent with their laws of motion, and obey market clearing.

[7] This is an "active" rule in the language of Leeper (1991), and a "non-Ricardian" policy in the language of Woodford (1995).

Definition 1. *Given (i) constant tax and transfer function $\tau^*(z)$ and government expenditure ratio x^*; and (ii) an initial distribution of households across asset and endowment shares $f_0(\omega, z)$, a real equilibrium is a collection of variables:*

$$\{v_t(a,z), c_t(a,z), g_t(a,z), a_t, b_t, r_t\}_{t\geq 0} \tag{20}$$

such that, for all $t \geq 0$:

1. *the value function $v_t(a,z)$ solves the HJB (8) and satisfies the boundedness condition (11)*
2. *the consumption function is defined by (9)*
3. *the distribution of asset levels $g_t(a,z)$ solve the KFE (12), with initial condition $g_0(\omega a_0, z) = f_0(\omega, z)$*
4. *aggregate household wealth a_t is given by $a_t = \sum_{z\in\mathcal{Z}} \int_{a\in\mathbb{R}_+} a g_t(a,z)\mathrm{d}a$*
5. *the path of government debt b_t satisfies the government budget constraint (18)*
6. *the asset market clears, $a_t = b_t$*

By Walras' law, asset market clearing implies that the goods market clearing condition is also satisfied:

$$\int_{j\in[0,1]} c_{jt}\mathrm{d}j = 1 - x^* \text{ for all } t \geq 0.$$

Price Level and Inflation Determination. Under our assumptions about monetary and fiscal policy, each real equilibrium implies a unique initial price level P_0 and a subsequent unique path of inflation π_t. These are determined as follows. Each real equilibrium contains an initial value of real government debt b_0. Since initial nominal debt B_0 is given, the initial price level is determined as

$$P_0 = \frac{B_0}{b_0}.$$

The path of inflation is then uniquely determined by the equilibrium path of real rates r_t and the nominal rate i_t as

$$\pi_t = i_t - r_t - g.$$

Different sequences of nominal rates $\{i_t\}_{t\geq 0}$ will therefore give rise to different paths for inflation $\{\pi_t\}_{t\geq 0}$ for the same sequence of real rates $\{r_t\}_{t\geq 0}$. For instance, if the monetary authority instead follows a lagged Taylor rule of the form

$$\mathrm{d}i_t = -\theta_m \left[i_t - \bar{i} - \phi_m(\pi_t - \bar{\pi})\right] \mathrm{d}t \tag{21}$$

then initial inflation is determined as $\pi_0 = i_0 - r_0 - g$ and subsequent inflation is determined as the unique forward solution to the ordinary differential equation

$$\mathrm{d}\pi_t = -\theta_m \left[\pi_t - \phi_m(\pi_t - \bar{\pi}) + r_t - (g - \bar{i})\right] \mathrm{d}t - \mathrm{d}r_t.$$

Depending on parameter configurations, prices and inflation may not remain bounded, but such paths are consistent with household optimality (and thus equilibrium). Our assumption of price flexibility thus allows us to isolate the impact of fiscal policy on price level determination and dynamics.[8] We therefore focus most of our analysis on the existence and uniqueness of real equilibria, with the understanding that whenever the real equilibrium is unique, so too is the price level and inflation.

The FTPL's Approach to Determinacy. To obtain a determinate initial price level, we rely on so-called active fiscal rules that give rise to equilibrium dynamics in which government debt grows fast enough to violate the household transversality condition for all values of initial real debt except one. Some economists argue that these fiscal rules are sensible institutional arrangements (Cochrane, 2023) and have large explanatory power in some historical episodes, such as the post-Covid inflation dynamics (Barro and Bianchi, 2023). Others have criticized them in the context of representative agent economies on the basis that they amount to off-equilibrium threats to "blow-up" the government budget (Kocherlakota and Phelan, 1999) and are not robust to small perturbations akin to the global games literature (Angeletos and Lian, 2023; Angeletos et al., 2023b, 2024). However, Angeletos et al. (2024)

[8]With nominal rigidities, both monetary and fiscal policy can affect inflation determinacy through their effect on output movements and debt accumulation. See, for example, Angeletos et al. (2024) for an analysis of these fiscal-monetary interactions in an analytical HANK economy, Mian et al. (2021a) for an analysis of these interactions in the presence of the ZLB, and Kaplan (2025) for an analysis of RANK models with bonds in utility.

and Kaplan (2025) show that in sticky price economies this fragility does not apply to models without Ricardian equivalence (including finite lives, bonds-in-utility, and imperfect risk sharing) which are the focus of our analysis. In addition, our results pertaining to the steady-states of our economy do not rely on active fiscal rules.

Extensions. Our results do not hinge on the specific formulation of the household problem, which has been constructed to convey our message about price level dynamics transparently. In Appendix F we extend the model to incorporate borrowing and long-term debt, in Not For Publication Appendix N.2 we replace our Poisson income process with a diffusion, and in Not For Publication Appendix N.6 we add endogenous labor supply and production.

3 A Useful Benchmark: Bonds in the Utility

In this section, we show how price level determination is obtained in a representative agent framework with bonds in utility (BIU). The BIU economy is useful because it shares two important properties with the heterogeneous agent economy: (i) failure of Ricardian equivalence, and (ii) an upward-sloping steady-state demand curve for household assets. Unlike in the heterogeneous agent economy, we can analytically characterize and illustrate the economy's equilibria.

Set Up. There is no idiosyncratic risk, so households receive a constant share of aggregate output, $\bar{z} = 1$. We assume that households derive instantaneous utility from their real asset holdings a_t according to some function $\xi(a)$. We impose that this function is twice-differentiable, strictly increasing and strictly concave, and that it satisfies an asymptotic boundedness condition $|\xi(a)| \leq Ca^{\varepsilon}$ for all a greater than some arbitrarily large threshold $\bar{a} > 0$, with $C > 0$ and $\varepsilon \in (0, 1)$.[9] To allow for the existence of equilibria with permanent deficits, we also impose that $\xi'(0) > \rho$ and finite. This ensures that households are willing to hold a strictly positive quantity of debt if the real rate is zero, which will be a key feature of our calibrated heterogeneous agent economy. Finally, to simplify the exposition, we assume $x_t = 0$ and $y_t = 1$, so that government spending and the growth rate of income are zero.

[9] This condition is to ensure that the real interest rate "grows fast enough" to violate transversality. It is satisfied for the commonly used logarithmic form used in, *e.g.*, Leeper (1991).

Equilibrium. Combining the household's Euler equation with market clearing ($c_t = 1$) implies that the real interest rate and quantity of assets are related by:

$$r_t = \rho - \xi'(a_t) \tag{22}$$

Consequently, the real rate is below the discount rate because of a convenience yield of holding debt, as captured by $\xi'(a) > 0$. Since $\xi''(a) < 0$, the gap $\rho - r_t$ is decreasing in a, which reflects that bonds might command a higher liquidity premium when they are in scarce supply. Moreover, there exists a finite lower bound $\underline{r} < 0$ below which households do not hold any debt. Along with the Euler equation (22), a necessary and sufficient condition for household optimality is the transversality condition:

$$\lim_{T\to\infty} e^{-\rho T} a_T = 0 \tag{23}$$

A *real equilibrium* of this economy is a path of real debt and interest rates $\{a_t, b_t, r_t\}_{t\geq 0}$ that satisfies the Euler equation (22), the transversality condition (23), the government debt evolution equation (18), and market clearing $a_t = b_t$.

3.1 Equilibrium with Surpluses

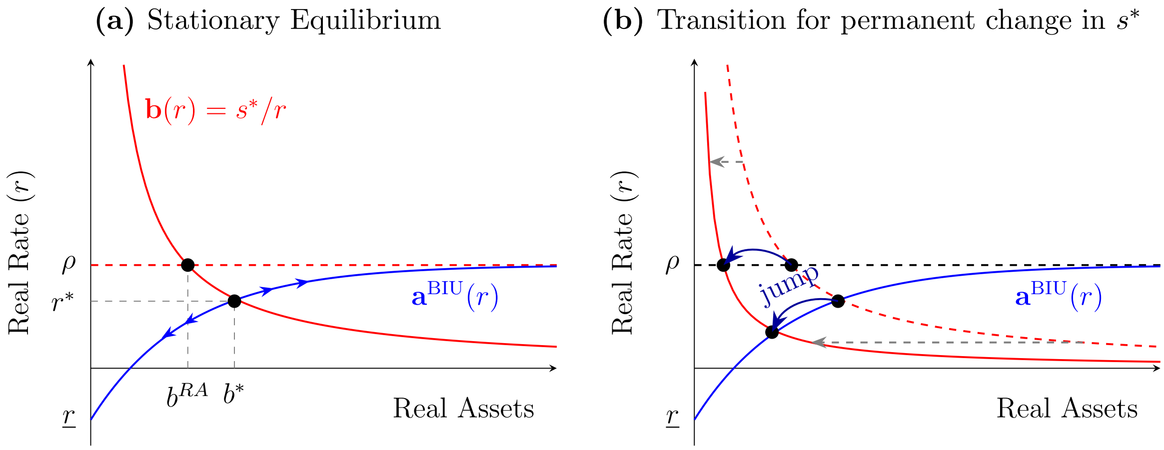
We first analyze price level determination when the government commits to a time-invariant transfer function τ^* that implements a constant level of surpluses $s^* > 0$.

Figure 1a plots the phase diagram of the system, defined by the Euler equation (22) and the government debt evolution equation (18). The curve $\mathbf{b}(r)$ defines a steady-state asset supply function, obtained by setting $\mathrm{d}b_t = 0$ in (18):

$$\mathbf{b}(r) = \frac{s^*}{r} \tag{24}$$

The curve $\mathbf{a}^{BIU}(r)$ plots the relationship between assets demanded by the household sector and real interest rates for an example function $\xi(a)$. Note that the Euler equation (22) holds both in and out of steady-state. For this reason, all equilibria lie on the one-dimensional manifold $\mathbf{a}^{BIU}(r)$ at all points in time. Since $\mathbf{a}^{BIU}(r)$ is increasing, $\mathbf{b}(r)$ is decreasing, and both functions satisfy Inada conditions, a unique

Figure 1: The Bond-in-Utility Model with Surpluses



Note: The left panel shows price level determinacy for the BIU economy when the government runs strictly positive surpluses. The arrows illustrate unstable dynamics for a fixed surplus rule. The right panel shows the transitional dynamics for a permanent reduction in surpluses in the BIU economy. Parameters: $\xi(a) = -15\exp(-0.4a)$, $s^* = 10$, $\rho = 4$, and the post-shift surplus is $s^{**} = 3$. The equilibrium of the RA economy corresponds to $r = \rho$.

steady-state of the system exists (given by the intersection of the two curves).

It is straightforward to show that the steady-state of the system is unstable: debt is increasing when it is above the steady-state, and decreasing when below steady-state. Paths with increasing debt can be ruled out as equilibria because they grow at a rate approaching ρ, and therefore violate the household's transversality condition (23). Paths with decreasing debt are ruled out since they violate the household's borrowing constraint in finite time. It thus follows that the stationary equilibrium in which real debt and real rates are constant is also the unique real equilibrium.

Proposition 1 formalizes this argument. The initial price level is determinate and inflation rate is constant for $t > 0$.

Proposition 1. *In the BIU economy with $s^* > 0$, a unique real equilibrium exists with*

$$P_0 = \frac{B_0}{b^*} \quad \text{and} \quad \pi_t = i^* - r^*$$

where b^* and r^* are the quantity of real debt and the real interest rate of the unique steady-state of the economy, respectively.

Proof. See Appendix B.1. □

The canonical representative agent (RA) model is a special case of the BIU economy when $\xi(a) = 0$, implying $r_t = \rho$ for all $t \geq 0$. The argument for uniqueness, and hence for a determinate price level, is identical. See Appendix A for a formal proof.

A Permanent Changes in s^*. Proposition 1 entails that in response to an unanticipated, permanent reduction in surpluses, the BIU economy instantaneously jumps to its new steady-state. Consider a fiscal authority that unexpectedly changes the tax function or government expenditures so that primary surpluses decline to $s^{**} = \Delta_s s^*$, with $\Delta_s \in (0,1)$. The new steady-state government bond supply function is $\mathbf{b}(r) = s^{**}/r$, which is displayed as a leftward shift in Figure 1b.

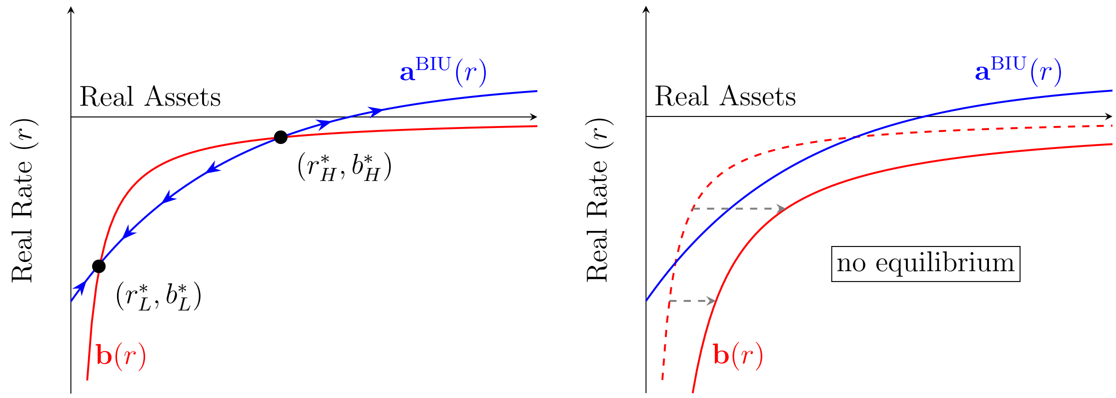
In both the RA and BIU economy, real rates and debt immediately jump to their new steady-state levels. In the RA economy, real rates remain constant while real debt falls by Δ_s%. This is achieved by a one-time upward jump in the price level from P_0 to P_0/Δ_s, with no subsequent change in inflation. The stock of nominal debt (which is predetermined) is unchanged, but real surpluses are reduced and thus the price level must jump to lower the real value of outstanding debt. In the BIU economy, both real rates and assets jump to a lower steady-state level, since asset demand is elastic: real debt is now scarcer, which increases the convenience yield and lowers the real interest rate. Hence, without a corresponding change in the nominal rate, inflation is permanently higher in the BIU economy.

A Fiscal Helicopter Drop. In the BIU economy, a one-time issuance of nominal debt leaves all real variables unchanged. If B_0 increases by Δ_B%, the price level also jumps by Δ_B%, leaving the level of real debt and subsequent inflation unaffected.

Differences Between BIU *vs.* HA Economies. The similarities between the BIU and the HA economy arise from the inelastic asset demand. But one important difference is that the HA model also features MPC heterogeneity and precautionary saving, which are absent from the BIU economy. Because of these additional forces, in the HA economy a fiscal helicopter drop or a permanent change in surpluses both give rise to transitional dynamics in which the real interest rate and inflation adjust slowly prior to reaching steady-state.

Figure 2: The Bond-in-Utility Model with Deficits

(a) Stationary Equilibria **(b)** Non-Existence



Note: The left panel shows that multiple stationary equilibria can exist when $s^* < 0$. The top steady-state is locally unstable, while the bottom state is stable. The right panel shows that no equilibrium exists if the level of deficits is sufficiently large. Parameter values: $\xi(a) = -40 * \exp(-0.3 * x)$, $s^* = -5$, $\rho = 2$, and post-shift deficit $s^* = -15$.

3.2 Equilibria with Deficits

The previous section showed that the price level is determinate when the government runs a strictly positive constant surplus.[10] In this section, we show that these insights do not generalize to the case in which a government runs a constant permanent deficit, $s^* < 0$. In this case, alternative fiscal rules are necessary to obtain uniqueness.

Existence and Non-Uniqueness. Figure 2a plots the household demand function $\mathbf{a}^{BIU}(r)$ and the asset supply function $\mathbf{b}(r)$ when $s^* < 0$. When $s^* < 0$, the household asset demand function is unchanged. However, the null-cline of the government budget constraint $\mathbf{b}(r)$ becomes an upward sloping hyperbola. Therefore, if a stationary equilibrium exists, there can be multiple stationary equilibria, as indicated by the two intersections of the asset supply and demand curves in the figure. In these equilibria, the real rate is negative, and the interest payments made by households to the government finance the permanent deficit. Nevertheless, debt is valued by households because of its convenience yield.

[10]When $s^* = 0$, our economy reduces to a standard monetary economy (Bewley, 1986) with a unique (finite) price level. For completeness, we treat this case in Appendix B.3.

These steady-states are Pareto ranked. The one featuring a high real debt Pareto dominates the other because it yields the same level of consumption (always constant in this endowment economy) but higher utility flow. In addition, a comparative statics exercise where we trace steady-states as surpluses shrink into deficits would yield the high-rate high-debt steady-state if we impose continuity with respect to r, a, and π.

A further implication of permanent deficits is that there exists a maximum level of deficits that is consistent with the existence of a stationary equilibrium where the price level is finite and government debt is valued—a result which echoes the literature's recent findings on debt sustainability (*e.g.*, Mehrotra and Sergeyev, 2021; Mian et al., 2021a). As the level of deficits increases, the government asset supply curve shifts downward to the right, as illustrated in Figure 2b. The maximum deficit is attained when the asset supply and demand curves are tangent to each other, which occurs at the point where the interest-rate elasticity of the steady-state household asset demand curve is equal to unity: $\mathbf{a}^{BIU\prime}(r)r/\mathbf{a}^{BIU}(r) = -1$. In the heterogeneous agent economy, this elasticity is determined by the strength of the precautionary insurance motive, which is endogenous and tied to observable features such as the degree of income risk and the progressivity of the tax and transfer system.

Non-uniqueness of the Price Level and Inflation. Since there can be multiple steady-state equilibria with $s^* < 0$, the argument underlying price level determinacy under surpluses is no longer valid. Figure 2a shows that the bottom steady-state, denoted by (r_L^*, b_L^*), is locally stable, while the top steady-state, denoted by (r_H^*, b_H^*), is locally unstable. To see this, observe that the linearized dynamics of the economy can be written as:

$$\dot{b} = \mathbf{r}(b)b - s^* \approx \left[\mathbf{r}'(b^*) + \frac{r^*}{b^*}\right](b - b^*) \tag{25}$$

for all b close to a steady-state b^*. The function $\mathbf{r}(b^*)$ is the inverse of the steady-state asset demand curve $\mathbf{a}^{BIU}(r)$. Moreover, it is easily seen that r^*/b^* is equal to $-1/\mathbf{b}'(r^*)$, which is the slope of the inverse of $\mathbf{b}(r)$ around the steady-state. With a surplus $s^* > 0$, $\mathbf{a}^{BIU}(r)$ is increasing and $\mathbf{b}(r)$ is decreasing so the steady-state is always locally unstable under a surplus. But with a deficit $s^* < 0$, both curves are increasing. Hence, only the top steady-state of Figure 2a is locally unstable (since

$\mathbf{a}^{BIU}(r)$ intersects $\mathbf{b}(r)$ from below), while the bottom steady-state is stable (since $\mathbf{a}^{BIU}(r)$ intersects $\mathbf{b}(r)$ from above).

For any initial value of real debt $b_0 \in (0, b_H^*]$ and associated initial price level, there is an equilibrium with a path for debt that converges to b_L^*. Consequently, there are multiple real equilibria and an indeterminate price level whenever there are multiple steady-states. We summarize these findings in the Proposition below.

Proposition 2. *In the BIU economy with $s^* < 0$, the following statements are true:*

1. *If no steady-state exists, then no real equilibria exist.*
2. *If two distinct steady-states exist, then there exists a continuum of real equilibria indexed by $b_0 \in (0, b_H^*]$.*

Proof. See Appendix B.2. □

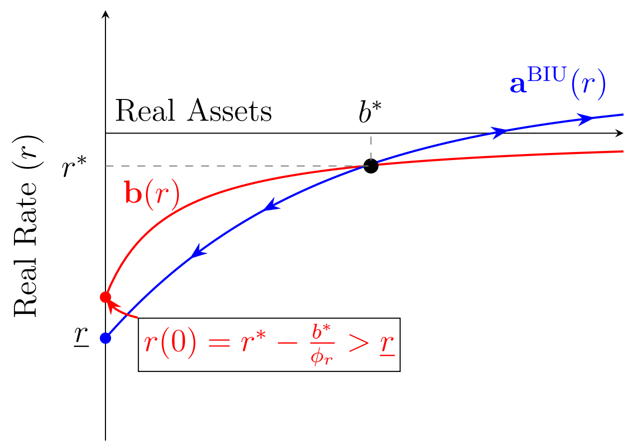
We note that in the special case of the canonical RA model, running persistent deficits cannot be an admissible equilibrium because $r_t = \rho > 0$ for all t.

Options for Price Level Determination with Deficits. While the price level is generally globally indeterminate with a fixed deficit, price level dynamics in the BIU economy are still *locally* determinate around the low-inflation steady-state (r_H^*, b_H^*). Thus, one can still obtain unique predictions for inflation if they are willing to assume that the economy eventually returns to the low-inflation steady-state after a shock. Note that this only requires that agents (or the central bank) coordinate on the *long-run* inflation expectations of the economy. Appendix C contains further details regarding this long-run anchoring.

There are also several alternative fiscal rules that can eliminate the locally stable steady-state and thus achieve global determinacy in the presence of deficits. For example, a government can implement a rule in which primary deficits are reduced when the real rate falls below its steady-state level r^*:

$$s_t = s^* + \phi_r(r_t - r^*), \quad \text{with } \phi_r < 0. \tag{26}$$

Figure 3: Uniqueness with a Real Rate Reaction Rule



Note: This figure shows that the government can implement a unique equilibrium when $s^* < 0$ under the real rate reaction rule of Equation (26). Parameter values: $\xi(a) = -40 * \exp(-0.3 * x)$, $s^* = -5$, and $\phi_r = -1$.

In this case, the null-clines of the government accumulation equation are defined by:

$$r(b) = \frac{(b^* - \phi_r)r^*}{b - \phi_r} \tag{27}$$

Figure 3 shows that this amounts to a leftward shift of the nullcline, which can eliminate the unstable steady-state and ensure global determinacy. In Appendix D, we show that rules in which surpluses respond to real debt deviations sufficiently aggressively, or rules in which the central bank targets the growth rate of nominal debt in the spirit of Hagedorn (2021) can also give rise to global determinacy.

4 Price Determination with Heterogeneous Agents

This section shows that in the HA economy price level determination inherits the qualitative properties of the BIU economy. However, because the wealth distribution enters as an infinite-dimensional state variable, equilibria do not lie on a one-dimensional manifold. For this reason, the characterization of equilibria is more challenging. In addition, short-run dynamics of inflation to a permanent change in surpluses or a fiscal helicopter drop differ from those of the BIU economy, due to wealth redistribution and precautionary saving. Appendix E contains additional details on the derivations.

4.1 Stationary Equilibria in the HA Economy

Under regularity conditions that are well understood, with a constant interest rate r and transfer function $\tau^*(z)$, the solution to (8) and (12) implies a finite upper bound $\bar{a}$ on the asset space, and a unique stationary distribution $g\left(a, z; r\right)$ (Bewley, 1986; Stokey et al., 1989; Aiyagari, 1994). We use this result to construct a function $\mathbf{a}(r)$ that maps different interest rates into the aggregate quantity of real assets held by households in the corresponding stationary distribution,

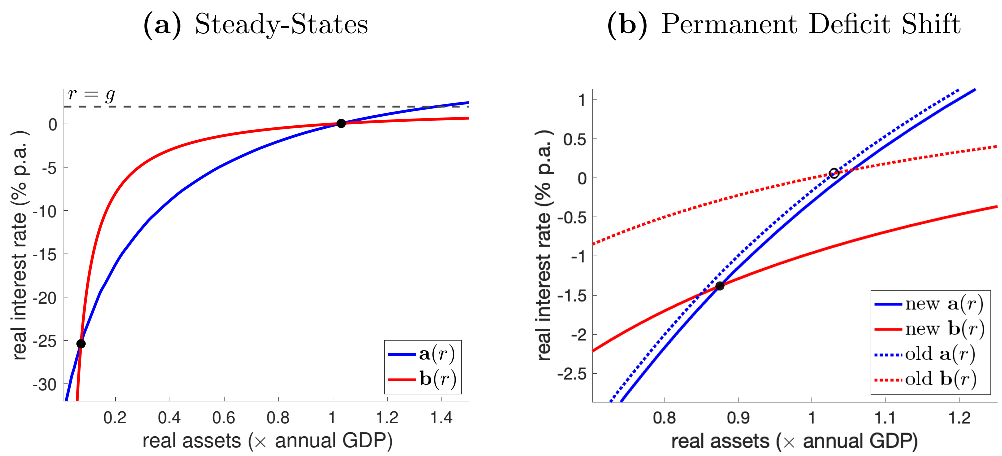
$$\mathbf{a}\left(r\right) := \sum_{z \in \mathcal{Z}} \int_{a \in \mathbb{R}_+} g(a, z; r) \mathrm{d}a$$

where $\lim_{r \to \rho} \mathbf{a}\left(r\right) = \infty$. In addition we will assume that the function $\mathbf{a}\left(r\right)$ is continuous, differentiable and strictly increasing. Achdou et al. (2022) identified sufficient conditions on the elasticity of intertemporal substitution and the borrowing limit ($\gamma \leq 1$ and $\underline{a} \geq 0$) for this to be true. Furthermore, in Appendix E.1 we show that—as for the BIU economy—there exists an interest rate $\underline{r}$ below which households do not hold any assets in the stationary distribution, so that $\mathbf{a}\left(r\right) = 0$ for all $r \leq \underline{r}$. In general, one can construct the utility function $\xi(a)$ so that steady-state asset demand with bonds in utility $\mathbf{a}^{BIU}(r)$ mimics the steady-state asset demand in the heterogeneous agent economy $\mathbf{a}(r)$.

Therefore, just as in the BIU economy, there is a unique stationary equilibrium with surpluses. With deficits, there are multiple steady-states as long as the level of deficits is not too large, because when r is sufficiently low households do not hold any debt (as illustrated in the right panel of Figure 2). While in principle more than two stationary equilibria could exist, in our experiments we have always found two (as long as the government implements deficits strictly below the maximum attainable one). In Appendix E.2 we prove that these two steady-state equilibria are Pareto ranked, with the high debt, high interest rate, low inflation steady-state delivering higher welfare for every household because of better consumption smoothing.[11]

[11] In addition, as in the BIU economy, the high debt steady-state is the selected one if we impose continuity (in r, a, π) on the comparative statics with respect to the size of surpluses.

Figure 4: Steady-States and Deficit Shifts in the HA Economy



Note: This figure shows how the steady-states of the heterogeneous agent economy respond to a persistent increase in deficits. The dotted lines plot the steady-state asset demand and supply curves prior to the increase. Relative to the BIU economy, both curves shift. See Table 1 for parameter values. The shock is a 1% reduction in proportional income taxes.

Comparative Statics. Unlike in the BIU economy, in the heterogeneous agent economy the effects of a permanent change in the level of surpluses depend on the specifics of how surpluses are changed, because they affect the stochastic process of households' disposable income and thus their desire to accumulate savings.

Consider a permanent increase in the primary deficit. If the larger deficit comes from more government consumption x, the $\mathbf{a}(r)$ curve will be unchanged. If, instead, it originates from larger lump-sum transfers, (i) wealth is redistributed toward high MPC households and, (ii) since the government provides more social insurance, all households will save less for precautionary reasons. As a result of these two forces, the $\mathbf{a}(r)$ curve will shift to the left.[12] Finally, a larger deficit due to lower proportional taxes increases both the mean and volatility of disposable income. If the second effect dominates, households increase their precautionary savings and the asset demand curve shifts to the right. In general, for a given increase in deficits, larger leftward shifts in the $\mathbf{a}(r)$ curve will reduce real debt and real rates by larger amounts.

[12] In this case the aggregate quantity of real debt is non-neutral because the borrowing constraint does not move one-to-one with the discounted present value of transfers (Aiyagari and McGrattan, 1998; Angeletos et al., 2023a).

Maximum Deficits. As in the BIU model, there exists a maximum level of attainable deficit such that debt is valued by households in equilibrium. A corollary of our discussion of comparative statics is that, unlike the BIU model, the composition of fiscal adjustment affects the largest sustainable deficit. In particular, fiscal changes that reduce households' demand for assets by shifting the $\mathbf{a}(r)$ to the left tighten this limit. These findings are related to the work of Reis (2021) and Mian et al. (2021a), who conclude that redistributive government transfers increase this limit in a model with rate-of-return risk and a BIU model, respectively.

Implications for Secular Stagnation. A recent literature argues that the secular decline of real rates observed in the US and other developed economies is due to rising income risk and inequality, accelerated by the sharp debt deleveraging that occurred after the 2008 financial crisis (Auclert and Rognlie, 2018; Eggertsson et al., 2019; Mian et al., 2021b). The argument is that higher inequality leads to a redistribution of income from the high-MPC poor to the low-MPC rich, which increases overall household demand for wealth. Similarly, more uninsured income risk or tighter borrowing limits create a stronger precautionary motive. These forces all manifest as an outward shift of the household asset demand function $\mathbf{a}(r)$. With a primary surplus and a positive real rate, such an outward shift in household asset demand indeed leads to a lower steady-state real rate. However, with a primary deficit and a negative real rate, these comparative statics are reversed when the economy starts in the low-inflation steady-state. An outward shift of the household asset demand function $\mathbf{a}(r)$ leads to a higher steady-state real rate. The reason is that households want to hold more government debt and so a less negative (i.e. higher) real rate is sufficient to finance the same level of deficits. This observation adds an important qualification to the commonly held view that shifts in the income distribution, income risk or deleveraging are candidate explanations for secular stagnation.

Our comparative statics in the heterogeneous agent model suggest an alternative explanation for secular stagnation: a permanent rise in fiscal deficits (or, equivalently, a permanent decline in surpluses) can account for a secular decline in real rates. This result is a distinguishing feature of the heterogeneous agent economy relative to the RA economy, where a permanent fall in surpluses is neutral on real rates.

4.2 Dynamic Equilibria in the HA Economy

To understand conditions for price level determinacy and to study impulse responses to unanticipated shocks, we need to understand the out of steady-state dynamics. Full derivations of the equations in this section are in Appendix E.

State-Space Representation for Heterogeneous Agent Economy. In the heterogeneous agent economy, the aggregate state variable is the distribution of real assets and productivity across households $g_t(a, z)$. It is useful to partition this distribution into two components, which we denote by $\Omega_t := \{f_t(\omega, z), b_t\}$

(i) $f_t(\omega, z)$: the joint distribution of household asset shares and endowment shares

(ii) b_t: the level of real government debt.

The reason for partitioning the aggregate state in this way is that the two components have different dynamic properties. The distribution $f_t(\omega, z)$ is backward-looking and cannot jump. The level of real debt is, instead, a jump variable. It can jump because different values of the initial price level P_0 revalue the outstanding stock of nominal bonds B_0, which is given. Partitioning in this way makes it clear that, although the household distribution $g_0(a, z)$ can jump, it can only jump along a single dimension such that the relative wealth holdings of each household remains unchanged. Using this state-space representation, we can write the consumption function $c_t(a, z)$ as $c(\omega, z, \Omega_t)$, where dependence on time is subsumed in the aggregate state.[13]

A Characterization of the Real Rate r_t. Using this representation, we can derive an expression that relates the real rate r_t to the aggregate state Ω_t. Let $c_{jt} := c(\omega_j, z_j, \Omega_t)$ denote the consumption of household j at time t. By aggregating the expected individual consumption growth $\mathbb{E}_{jt}[\mathrm{d}c_{jt}]$, we arrive at

$$\underbrace{\frac{1}{\gamma}\mathcal{C}_t^u(r_t-\rho)}_{\text{intertemporal subst.}} + \underbrace{\frac{\gamma+1}{2}\mathcal{C}_t^u\tilde{\mathbb{E}}_t^{\mathrm{u}}\left[\sum_{z'}\lambda_{z_jz'}\left(\frac{c(\omega_j,z',\Omega_t)}{c_{jt}}-1\right)^2\right]}_{\text{precautionary saving due to prudence}} + \underbrace{\mathcal{C}_t^{\mathrm{c}}\tilde{\mathbb{E}}_t^{\mathrm{c}}\left[\sum_{z'}\lambda_{z_jz'}\left(\frac{c(\omega_j,z',\Omega_t)}{c_{jt}}-1\right)\right]}_{\text{precautionary saving due to constraints}} \approx 0 \tag{28}$$

[13] In our quantitative analysis, we consider unanticipated shocks to various exogenous parameters. In these cases, the state space Ω_t is expanded to include the paths for the exogenous driving processes.

The expectation operators $\tilde{\mathbb{E}}_t^{\mathrm{u}}$ and $\tilde{\mathbb{E}}_t^{\mathrm{c}}$ are, respectively, the consumption-weighted mean across unconstrained and constrained households. Similarly, $\mathcal{C}_t^{\mathrm{u}}$ and $\mathcal{C}_t^{\mathrm{c}}$ denote total consumption of these two groups. The $\approx$ sign is the result of a second-order approximation (with respect to z) in one of the terms. Appendices E.3 and E.4 contain details of this derivation, including the exact, but somewhat less intuitive, formulation.

Equation (28) balances three forces driving changes in aggregate consumption that must net out to zero in an endowment economy. The first term is an intertemporal substitution motive for saving. The second term is the aggregate precautionary savings motive due to prudence ($\gamma + 1$ is the coefficient of relative prudence), a force which is only active for unconstrained households. The final term reflects the aggregate precautionary saving motive due to occasionally binding borrowing constraints. The more severe are constraints, the larger the share of consumption of constrained households, and the higher their expected consumption growth. In equilibrium, the interest rate is set so that the negative intertemporal substitution motive exactly offsets the two precautionary saving motives.

Local Saddle-path Stability. Equation (28) is useful because it implies that we can define a time-invariant functional from Ω_t to r_t that holds at all times in equilibrium:

$$r_t = \mathbf{r}\left[\Omega_t\right]. \tag{29}$$

We can derive the dynamics of the aggregate state Ω_t by expressing the Kolmogorov Forward Equation (12) in terms of asset shares, and substituting the real rate functional (28) into the government budget constraint (18):

$$\begin{aligned} \partial_t f_t(\omega, z) &= -\partial_\omega \left[f_t(\omega, z) \frac{1}{b_t} \left\{ z - \tau^*(z) - c(\omega, z, \Omega_t) + s^* \omega \right\} \right] \\ &\quad - f_t(\omega, z) \sum_{z' \neq z} \lambda_{zz'} + \sum_{z' \neq z} \lambda_{z'z} f_t(\omega, z') \end{aligned} \tag{30}$$

$$\frac{\mathrm{d}b_t}{\mathrm{d}t} = \mathbf{r}\left[\Omega_t\right] b_t - s^* \tag{31}$$

Since this system is comprised of a one-dimensional jump component b_t and an infinite dimensional backward looking component $f_t(\omega, z)$, the relevant notion for local price determinacy is saddle-path stability. This requires that, given an initial distribution $f_0(\omega, z)$ in the vicinity of some steady-state $f^*(\omega, z)$, there is a unique initial value for the jump variable b_0 and unique subsequent paths of the aggregate state Ω_t such that the economy converges to $\Omega^* = \{f^*(\omega, z), b^*\}$. Mathematically, this holds when the linearization of the PDE system around Ω^* has exactly one eigenvalue with a positive real part, while the remaining elements of the spectrum have non-positive real parts.

Discretized Economy. Analyzing saddle path stability for the full continuum economy is challenging because of the infinite dimensional asset share distribution. However, in numerical explorations of discretized versions of this economy, we have found the steady-state to be saddle-path stable when $s^* > 0$, and only the low-inflation steady-state to be saddle-path stable when $s^* < 0$. Here we offer some intuition for local saddle-path stability in this discretized economy.

Consider a discretization of $f(\omega, z)$ on a grid for relative wealth shares and endowments of size $N = N_\omega \times N_z$, and denote it by the $N \times 1$ vector $\mathfrak{f}$. The finite difference approximation of the PDE system (30) is given by the system of $N + 1$ ODEs

$$\frac{\mathrm{d}\mathfrak{f}}{\mathrm{d}t} = \mathbf{A}_\omega \left[\mathfrak{f}_t, b_t\right]^T \mathfrak{f}_t + \mathbf{A}_z^T \mathfrak{f}_t \tag{32}$$

$$\frac{\mathrm{d}b}{\mathrm{d}t} = \mathbf{r}\left[\mathfrak{f}_t, b_t\right] b_t - s^* \tag{33}$$

The $N \times N$ matrices $\mathbf{A}_\omega \left[\mathfrak{f}, b\right]^T$ and $\mathbf{A}_z^T$ are upwind finite difference approximations to the two linear operators that comprise the KFE for $(\{\omega_n\}_{n=1}^N, z)$.[14]

The dependence of $\mathbf{A}_\omega \left[\mathfrak{f}_t, b_t\right]^T$ on the distribution $\mathfrak{f}_t$ and real debt b_t arises for two reasons. First, the total quantity of real debt directly effects the evolution of households' wealth shares. For a household with a a given level of savings (income net of taxes and consumption), its wealth *share* moves by less when when the aggregate quantity of real debt is larger. This is captured by the denominator in Equation (30). Second, there are further general equilibrium effects on consumption because of

[14]The transposes reflect the fact the operators in the KFE (30) are the adjoint of the operators in the HJB equation, and those matrices in (32) are first constructed from the finite difference approximation to the HJB equation (8). See Appendix E.5 for more details.

future interest rate dynamics (which in turn affect the evolution of real debt). This is reflected in the dependence of the consumption function on the aggregate state Ω_t in the third argument of the consumption decision rule.

Linearizing the discretized system (32) around a steady-state $(\mathfrak{f}^*, b^*)$, the local dynamics are approximately

$$\begin{pmatrix} \frac{\mathrm{d}\mathfrak{f}}{\mathrm{d}t} \\ \frac{\mathrm{d}b}{\mathrm{d}t} \end{pmatrix} \approx \begin{pmatrix} \mathbf{A}_\omega^{*T} + \mathbf{A}_z^T & \nabla_b \mathbf{A}_\omega^T [\mathfrak{f}^*, b^*] \\ 0 & b^* \left\{ \partial_b \mathbf{r} [\mathfrak{f}^*, b^*] - \left(-\frac{r^*}{b^*}\right) \right\} \end{pmatrix} \begin{pmatrix} \mathfrak{f}_t - \mathfrak{f}^* \\ b_t - b^* \end{pmatrix} \tag{34}$$

where the term $\nabla_b \mathbf{A}_\omega^T [\mathfrak{f}^*, b^*]$ is the $N_\omega \times 1$ vector of derivatives of $\mathbf{A}_\omega^{*T}$ with respect to real debt b. The approximation in (34) refers to the zero in the bottom left element of the Jacobian. The intuitive reasoning that follows requires this term to be small only relative to the term in the bottom right element of the Jacobian. This requirement means that, around the steady-state, the dynamics of real government debt are more sensitive to changes in the *level* of real debt, holding the distribution of asset shares constant, than to shifts in the *distribution* of asset shares, holding the level of real debt constant.[15]

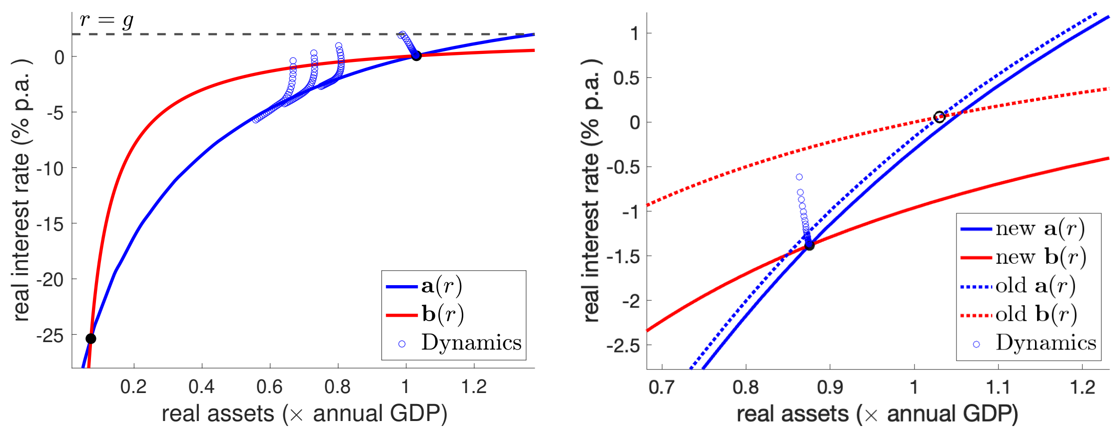
In this case, the Jacobian is approximately upper-block triangular, allowing us to sign the eigenvalues of the full system: the upper-left block $\mathbf{A}_\omega^{*T} + \mathbf{A}_z{}^T$ is an irreducible transition rate matrix and so has a single zero eigenvalue and remaining negative eigenvalues. The sign of the remaining eigenvalue is given by the sign of the lower-right scalar $\partial_b \mathbf{r} [\mathfrak{f}^*, b^*]\, b^* + \mathbf{r} [\mathfrak{f}^*, b^*]$. Under our working approximation, the first term is the inverse of the derivative of the steady-state household asset demand curve, multiplied by the level of steady-state assets. The second term is the steady-state interest rate. Hence, the condition for local stability is determined by the sign of

$$\frac{\mathrm{d}b_t}{\mathrm{d}t} \approx b^* \left((\partial_r \mathbf{a}(r^*))^{-1} - (\partial_r \mathbf{b}(r^*))^{-1} \right) \tag{35}$$

[15]This assumption might appear at odds with our substantive messages that emphasize changes in the distribution of real wealth as a quantitatively important factor in driving inflation and price level dynamics. However, as our simulations confirm, these are not contradictory: the feedback from the distribution of shares to the debt dynamics are large enough to be quantitatively meaningful, but would need to be orders of magnitude larger to alter the qualitative features of the dynamic system.

Figure 5: Dynamics in the Heterogeneous Agent Economy

(a) Illustrating Stability **(b)** Permanent Deficit Dynamics



Note: The left panel illustrates saddle-path stability for the low-inflation steady-state and stability for the high inflation steady-state. The initial share distribution $f_0(\omega, z)$ that gives rise to dynamics is obtained by perturbing the steady-state wealth shares ω_{ss} to $(\omega_{ss} + 0.17)/1.17$. This is the distribution that arises when all households in steady-state are given a helicopter drop equal to 17% of their outstanding nominal wealth. The circular markers (from light to dark as time goes by) illustrate the economy's transition in response to this unanticipated shock. The right panel illustrates the transitional dynamics in response to a permanent deficit expansion that arises from a 1% reduction in proportional taxes. The dotted lines depict the pre-shock curves. See Table 1 for parameter values.

where we have used the fact that $-r^*/b^* = (\partial_r \mathbf{b}(r^*))^{-1}$. The stability properties of each steady-state therefore coincide with the BIU economy. With a primary surplus, both the steady-state asset supply and demand curves are increasing in r, hence the remaining eigenvalue is strictly positive and the economy is saddle-path stable.

With a primary deficit, only the low inflation steady-state is saddle-path stable, since $\mathbf{a}(r)$ intersects $\mathbf{b}(r)$ from below. The high-inflation steady-state is locally stable, since $\mathbf{a}(r)$ intersects $\mathbf{b}(r)$ from above. Simulations confirm these properties.

Illustrating Saddle-path Stability. Figure 5a illustrates these dynamics. For a given initial distribution $f_0(\omega, z) \neq f^*(\omega, z)$, there is a unique equilibrium converging to (b_H^*, r_H^*) and a continuum of equilibria converging to (b_L^*, r_L^*), indexed by the initial level of real debt b_0. Consequently, the price level and inflation are not pinned down without additional assumptions that rule out almost all of these equilibria. Because the top equilibrium is saddle-path stable, there is a lower bound on the initial price level that is consistent with equilibrium. This minimum initial price level is given by $P_0 = \frac{B_0}{b_0}$, where b_0 is the unique initial value of real debt for which the economy converges to the top saddle-path stable equilibrium. Thus, the conditions for price level determinacy are qualitatively similar to the BIU economy. Note also that the paths converging to the bottom steady-state eventually lie on the $\mathbf{a}(r)$ curve, as in the BIU economy.

Dynamics After a Permanent Increase in the Primary Deficit. Figure 5b revisits the experiment in which the fiscal authority unexpectedly and permanently increases its primary deficit by reducing proportional taxes. As discussed above, this shifts both $\mathbf{a}(r)$ and $\mathbf{b}(r)$ by altering the degree of risk-sharing in the economy. In particular, the $\mathbf{a}(r)$ curve shifts to the right because the volatility of households' (post-tax) earnings is now higher. Moreover, unlike in the BIU economy, the economy does not jump immediately to the new steady-state. Instead, saddle-path dynamics imply that on impact of the change there is a one-time jump in the level of real debt to the unique value of b_0 that is consistent with non-explosive dynamics, which then determines a unique r_0 through the real rate functional (28). Both real debt and the real rate then converge to their new, lower, steady-state levels. This figure also illustrates how a long-lasting sequence of gradual deficit expansions can be a source of secular stagnation in real rates.

Ruling Out Explosive Equilibria. The preceding discussion provided intuition for local determinacy of the price level for different steady-states, but did not analyze global determinacy. As in the BIU economy, any paths for b_t that diverge in either direction are not consistent with equilibrium because they involve eventual violation of either the borrowing constraint or household transversality. See Appendix E.6 for a formal statement.[16]

[16] Because of the high dimensionality of the heterogeneous agent economy, a full global analysis would require allowing for the possibility of additional bounded non-stationary equilibria, such as limit cycles or chaotic dynamics. A complete global analysis is beyond the scope of this paper. See Acharya and Benhabib (2024) for an example of such dynamics in a related model that can be reduced to a three dimensional system.

Eliminating the Stable Steady-State. As shown in Section 3.2 in the context of the BIU model, we can disregard the high inflation steady-state by adopting alternative fiscal rules or coordinating long-run inflation expectations. Hence in the quantitative exercises in the following section, we focus on the saddle-path stable dynamics around the low-inflation steady-state. We also explore the robustness of our quantitative exercises to these alternative fiscal rules.

Alternative Approaches to Check Determinacy. The preceding analysis of local dynamics was based on a discrete approximation to a state-space representation of the economy. Apart from the fact that we do not offer a formal proof of determinacy even for this discretized economy, there are at least two limitations to this approach. First, by focusing on a state-space representation with the distribution of wealth $(\mathfrak{f}_t, b_t)$ as the aggregate state, we exclude the possibility of additional equilibria in an expanded state space. An alternative approach would be to write the consumption function in sequence space and employ the winding number criterion developed in Auclert et al. (2023) or Hagedorn (2023) to establish conditions for determinacy. Our numerical solutions are based on a sequence space approach and the equilibria we have found confirm the relevant dynamics around each of the two steady-states. Second, since the underlying continuum economy is described by a PDE rather than a system of ODEs, in general one cannot conclude that the local dynamic properties of a large but finite dimensional approximation to the system of ODEs coincide with those of PDE of the underlying infinite dimensional economy (using either the eigenvalue or winding number approach).[17]

A Note About Steady-State Comparisons. The preceding analysis of out-of-steady-state dynamics and price level determinacy assumed "active" fiscal rules (Leeper, 1991). However, we note that the comparative statics across the economy's steady-state equilibria do not rely on this specification of policy, and hold for any fiscal rule that implements a higher or lower steady-state primary deficit. These comparative statics results are also robust to the presence of nominal rigidities.

[17] See Bilal (2023) for a discussion. Saddle-path stability of the PDE system requires all but one of the elements of the spectrum to be bounded below zero. We have verified that all but one of the eigenvalues of the discretized ODE system are negative, but one can never be sure they are bounded below zero via numerical simulations.

5 Quantitative Exercises with Persistent Deficits

In this section we describe various quantitative experiments for a calibrated version of the model with persistent deficits in order to illustrate the role of redistribution and precautionary saving in shaping price level dynamics. We incorporate the following two extensions of the baseline model.

Extension I: Unsecured Household Credit. We allow for a non-zero borrowing limit. This permits nominal positions to be negative, thereby allowing some households to experience a positive wealth effect from an unanticipated rise in the price level, as in Doepke and Schneider (2006). We assume that households can borrow up to a fixed limit that is denominated in real terms. We interpret it as unsecured borrowing, such as credit card debt, and impose an exogenous wedge between borrowing and saving rates. See Appendix F.1 for details.

Extension II: Long-Term Debt. We assume that the government issues long-term debt with a constant maturity rate. Appendix F.2 describes this extension. The switch to long-term debt has no effect on the preceding analysis of price level determination. However, as shown by Sims (2011) and Cochrane (2018), debt duration plays a key role in the dynamics of inflation after unanticipated changes in the nominal interest rate. We explore this mechanism in Appendix N.8 where we consider monetary policy rules beyond an interest rate peg.

5.1 Parameterization

Preferences. We set the elasticity of intertemporal substitution γ to 1 so that households have log utility. We choose the discount rate ρ to match an annual debt-to-GDP ratio of 1.03 in the low inflation steady-state. This target corresponds to the average debt-to-GDP ratio in US data for the years 2014-2019 and implies a calibrated annual discount rate of 5.3%.

Endowment Process. We assume an annual aggregate real growth rate g of 2%, which was the US per-capita average over the post-war period.[18] Idiosyncratic en-

[18] See Series A939RX0Q048SBEA_PC1 from FRED, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org.

Table 1: Calibrated parameter values and targets.

Parameter	Value	Target
Preferences		
γ Inverse EIS	1	
ρ Discount rate	5.3% p.a.	debt-to-annual GDP ratio of 1.03
Income Process		
g Real output growth	2.0% p.a.	average growth rate post-war
λ Arrival rate of earnings shocks	1.0 p.a.	
σ St. Dev. of log quarterly earnings	1.2	
Household Borrowing		
$\underline{a}$ Borrowing limit	$\$18,600$	70% of quarterly household earnings
$r^b - r$ Borrowing wedge	16% p.a.	average rate on credit card debt
Tax and Transfers: $\tau(z) = \tau_0 - \tau_1 * z$		
τ_1 Proportional tax rate	32%	personal taxes / labor income
τ_0 Lump sum transfer	20% of GDP	deficit: $s^* = -2.0\%$
Government Consumption		
x_t Government consumption	14%	gov. consumption / GDP
Government Debt		
δ Maturity rate of government debt	20% p.a.	average duration of 5 years
Monetary Policy		
i Nominal rate	2.6%	long-run inflation of 2.5%

dowment shares follow an $N_z = 5$ state process, with switching rates chosen so that income shocks arrive on average once per year and the endowment process generates a standard deviation of log quarterly earnings of 1.08, in line with US micro data.[19]

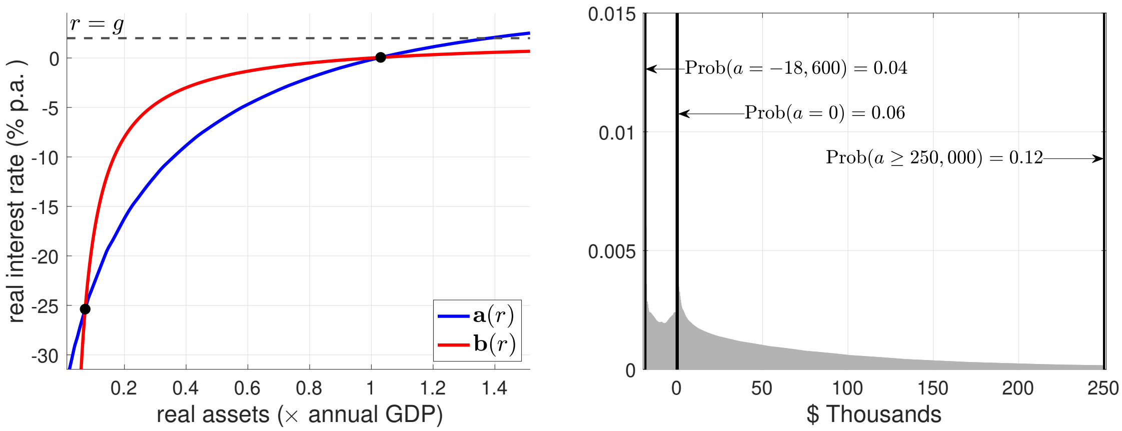
Household Borrowing. We set the borrowing limit $\underline{a}$ to \$18,600, which is approximately 70% of average quarterly household earnings to match the median credit card limit for working-age population in the Survey of Consumer Finances (SCF) (Kaplan and Violante, 2014). We set the wedge between the interest rates on borrowing and saving to 16% p.a., based on typical interest rates on unsecured credit card debt.[20] Because of this exogenous wedge, the real borrowing rate is positive, and the natural borrowing limit is finite and exceeds the calibrated limit.

[19] See, for example, the Global Repository of Income Dynamics (GRID), `https://www.grid-database.org/`.

[20] See Table Consumer Credit - G19, Federal Reserve Board, `https://www.federalreserve.gov/releases/g19/current/`.

Figure 6: Calibrated steady-states and wealth distribution

(a) Calibrated steady-states **(b)** Wealth distribution



Taxes, Transfers, and Government Consumption. The tax and transfer system consists of a lump-sum transfer and proportional tax,

$$\tau(z) = -\tau_0 + \tau_1 z.$$

We set the proportional tax rate τ_1 to 32% to match the ratio of personal taxes and social insurance contributions to total labor income (NIPA Table 2.9) for 2014-2019. We then set the total of lump-sum transfers and government consumption ($\tau_0 + x_t$) at 34.0% of aggregate output to generate a primary deficit s^* of -2.0% of GDP, the average for the US over that period.[21] We also set x_t at 14% to match the ratio of government consumption expenditures to GDP over 2014-2019.

Government Debt. We assume that 20% of outstanding government debt matures each year to match a weighted average duration of 5 years (US Treasury). Given our target debt-to-GDP ratio of 103%, and primary deficit of 2.0%, the implied steady-state real interest rate equals $\frac{s^*}{B^*} + g = \frac{-0.02}{1.03} + 0.02 \approx 0.1\%$ p.a.

Monetary Policy. We assume that the central bank pegs the nominal rate at 2.6% p.a., consistent with the average federal funds rate in the year leading up to the pandemic. With a real interest rate of 0.1%, the implied inflation rate is 2.5% p.a.

[21] For debt, see series GFDEGDQ188S from FRED. For deficits, we use the CBO's historical budget data. We calculate primary deficits by subtracting interest rate expenses on debt.

Representative Agent Model. In the representative agent economy we set the discount rate to replicate the same steady-state debt-to-GDP ratio as in the heterogeneous agent economy. However, since this economy does not admit a steady-state with persistent deficits, we assume an annual surplus-to-GDP ratio of 2.0%, which is obtained by a proportional tax rate of $\tau_1 = 0.02$. The equilibrium interest rate is therefore $\frac{s^*}{B^*} + g = \frac{0.02}{1.03} + 0.02 \approx 3.9\%$ p.a. We also adjust the nominal interest rate so that the inflation rate is the same in the two economies.

5.2 Properties of Steady-States

Figure 6a displays the two stationary equilibria implied by our calibration of the HA model. In line with our targets, the low inflation steady-state has an annual debt-to-GDP ratio of 103% and an annual inflation rate of 2.5%. The high inflation steady-state has an annual debt-to-GDP ratio of approximately 7.0%, and an annual inflation rate of 28.0%. In what follows, we focus on the low-inflation steady-state due to its saddle-path stability. To justify focusing attention on the unique saddle-path equilibrium under our permanent deficit rule, we are implicitly appealing to long-run inflation anchoring (see Appendix C).

Wealth and MPC Distribution. Figure 6b and Table 2 show that the model is broadly consistent with the distribution of liquid wealth in the 2019 SCF.[22] Expressed in 2019 dollars, mean and median household wealth in the model are $\$109,400$ and $\$36,700$ respectively. 19% of households have negative wealth and 28% of households have less than $\$1,000$. The average quarterly MPC in the model is 13.6%, with the highest MPCs among the low-income households that either have close to zero wealth and so are near a kink in their budget constraint, or have substantial negative wealth and so are close to the borrowing limit.[23]

[22]Our definition of liquid wealth includes money market, checkings, savings, and call accounts, as well as directly held mutual funds, stocks and bonds, minus credit card and uncollateralized debt. We exclude the top 1% of households in the SCF by liquid wealth because of the well-known difficulties in matching the right-tail of the wealth distribution in this class of models.

[23]Not For Publication Appendix N.4 contains additional details on the distributions of wealth and marginal propensities to consume in the model.

Table 2

Mean liquid assets	Data	Model
Mean assets	$\$116,000$	$\$109,400$
Frac. with $a < \$0$	21%	19%
Frac. with $a < \$1,000$	37%	28%

Note: Moments of the wealth distribution in the model and the data. Monetary values expressed in 2019 dollars. Data is from the 2019 Survey of Consumer Finances (SCF) with the top 1% of households by liquid wealth are excluded. See the main text for the definition of liquid assets in the data.

Maximum Sustainable Deficit. As discussed in Section 4.1, the maximum level of permanent deficits such that households still value holding debt depends on how the fiscal deficit is expanded. Under our calibration, increasing government consumption yields a maximum deficit of 4.34% of output, a 117% rise from the baseline steady-state value of 2.0%. Raising transfers yields a maximum deficit of 4.17%, while lowering taxes allows the government to run a maximum deficit of 4.61%.

Lower proportional tax rates are, in general, associated with higher maximum steady-state deficits because they increase the volatility of disposable earnings. Households therefore bear more uninsured idiosyncratic income risk which raises their overall precautionary demand for safe liquid assets. For a given interest rate r, households are willing to hold more government bonds if they bear more idiosyncratic risk, giving the government more room to expand its deficit. Graphically, a lower value for τ_1 induces an outward shift in the steady-state household asset demand curve. The same logic, with signs reversed, applies to an expansion of lump-sum transfers because they reduce the volatility of net earnings.

The roles of redistribution and precautionary saving are quantitatively important. For example, if we shut down proportional taxes ($\tau_1 = 0\%$), the maximum sustainable deficit that can be achieved by expanding transfers is 9.30%, almost five times as large as in our baseline. For similar reasons, when households are prohibited from borrowing and must hold liquid wealth to smooth shocks, the maximum sustainable deficit rises to 5.80%. A key policy lesson from these experiments is that reforms that loosen credit, make tax and transfer systems more progressive, or provide more public insurance to households reduce future fiscal space available to the government (see also Reis (2021) and Mian et al. (2021a) for a discussion of these forces in alternative frameworks). These reforms restrict the government's ability to expand deficits or cut surpluses, and therefore may constrain the use of expansionary fiscal policy to respond to adverse aggregate shocks.

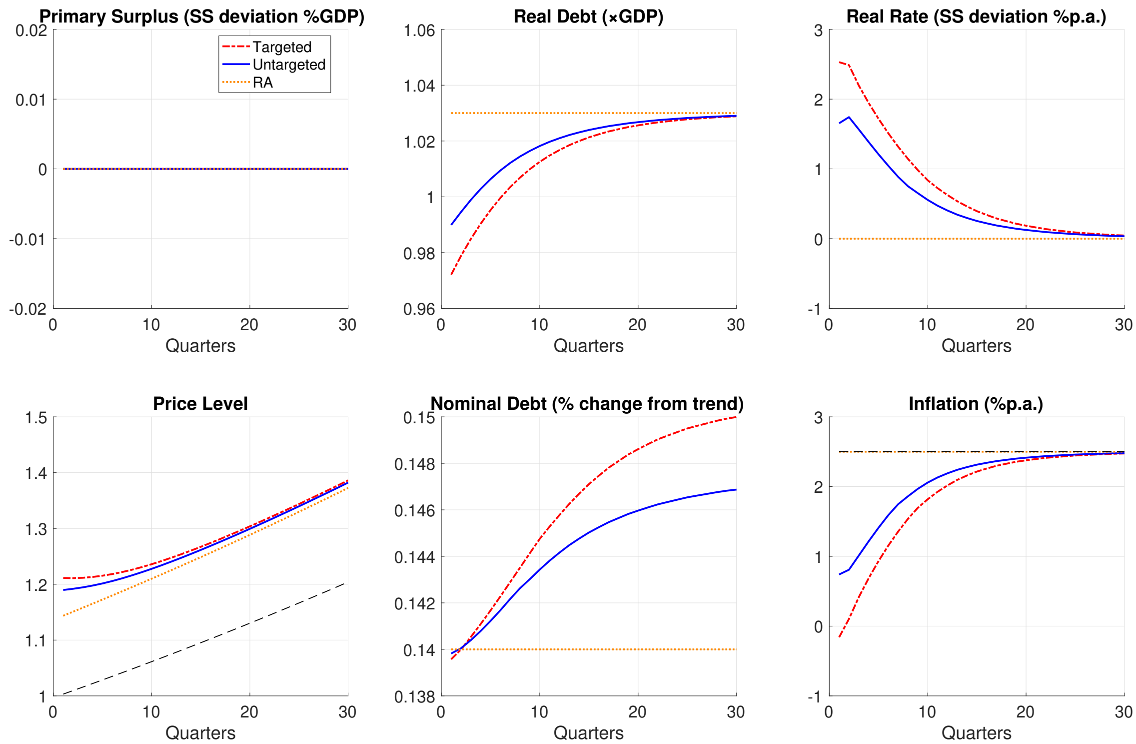
Robustness of Maximal Deficits. These calculations of the maximum sustainable deficit depend on the household discount rate ρ, a parameter that is inferred from data on aggregate wealth given the remaining model parameters, and is difficult to measure directly. Appendix G reports a robustness analysis on the maximum deficit where the discount rate is progressively lowered. Lower discount rates are associated with larger maximum sustainable deficits. In the extreme case where $\rho \simeq 0$ (but still strictly positive to make the household problem well defined), the maximum deficits that can be achieved by increasing lump-sum transfers is approximately 6.5%.

5.3 Fiscal Helicopter Drop

Our first numerical experiment is inspired by the experience of the US and other developed countries in the wake of the COVID-19 shock. In response to the disruptions caused by the pandemic, the US issued a large quantity of additional government debt and distributed much of the proceeds to households. We capture the core features of this fiscal helicopter drop by simulating an unexpected one-time issuance of nominal debt equal to 14% of initial outstanding government liabilities (equivalent to the observed 14% rise in US nominal debt in 2020), which is distributed as a one-time lump-sum transfer to households. We consider two versions of this policy: one where transfers are distributed uniformly and one where transfers are distributed only to households in the bottom 60% of the wealth distribution, in line with the actual US experience.

Aggregate Effect of Fiscal Helicopter Drop. The effects of the fiscal helicopter drop are displayed in Figure 7. Since there are no changes to primary surpluses or any other structural parameters, the helicopter drop has no permanent real effects: the household and government null-clines are unchanged, and the economy converges back to its initial steady-state.

Figure 7

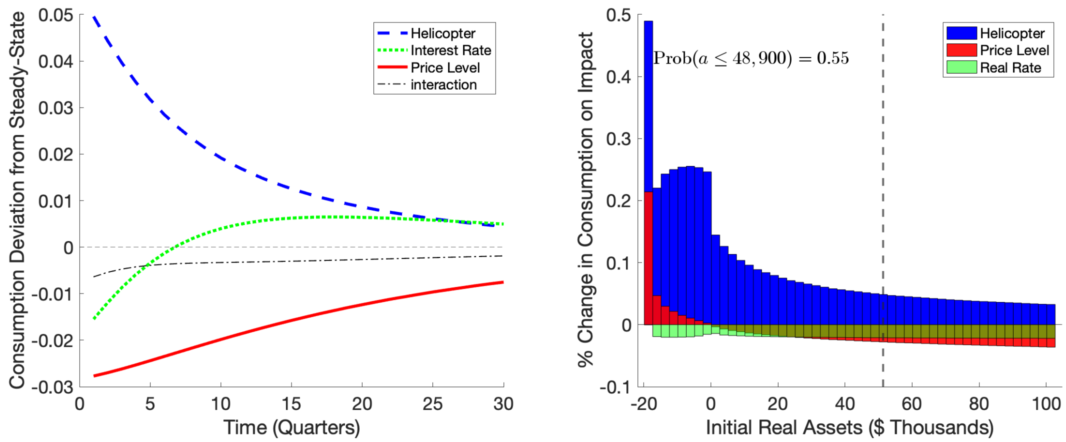


Note: This figure plots impulse responses to a targeted and untargeted helicopter drop, aggregated at the quarterly frequency. The helicopter drop is a one-time issuance of 14% of total government nominal debt outstanding at $t = 0$. Only households in the bottom 60% of the wealth distribution receive the issuance in the targeted experiment (dashed red line). The orange line plots dynamics in the representative agent (RA) model. The dashed black line plots the pre-shock trend.

In the representative agent version of this economy, which is shown by the orange dotted line labelled "RA" in Figure 7, convergence is instantaneous. The jump in the price level exactly offsets the new issuance of nominal debt so that the level of real debt remains constant and there are no further effects of the shocks.[24] In the heterogeneous-agent model, however, there are transitional dynamics. The initial jump in the price level (bottom-left panel of Figure 7) is about 19%, which more than offsets the 14% rise in nominal debt.

[24]The initial price jump in Figure 7 is slightly more than 14% because in this and other figures, we plot impulse response functions aggregated to a quarterly frequency.

Figure 8



Note: This figure decomposes the effect of the helicopter drop on consumption into its general equilibrium sub-components. The left panel depicts how each sub-component affects aggregate consumption over time in isolation. The right panel depicts the effect of each sub-component on initial consumption across the wealth distribution. The dashed black line on the right panel delineates households that experienced initial consumption gains and losses as a result of the helicopter drop in 2019 US dollars.

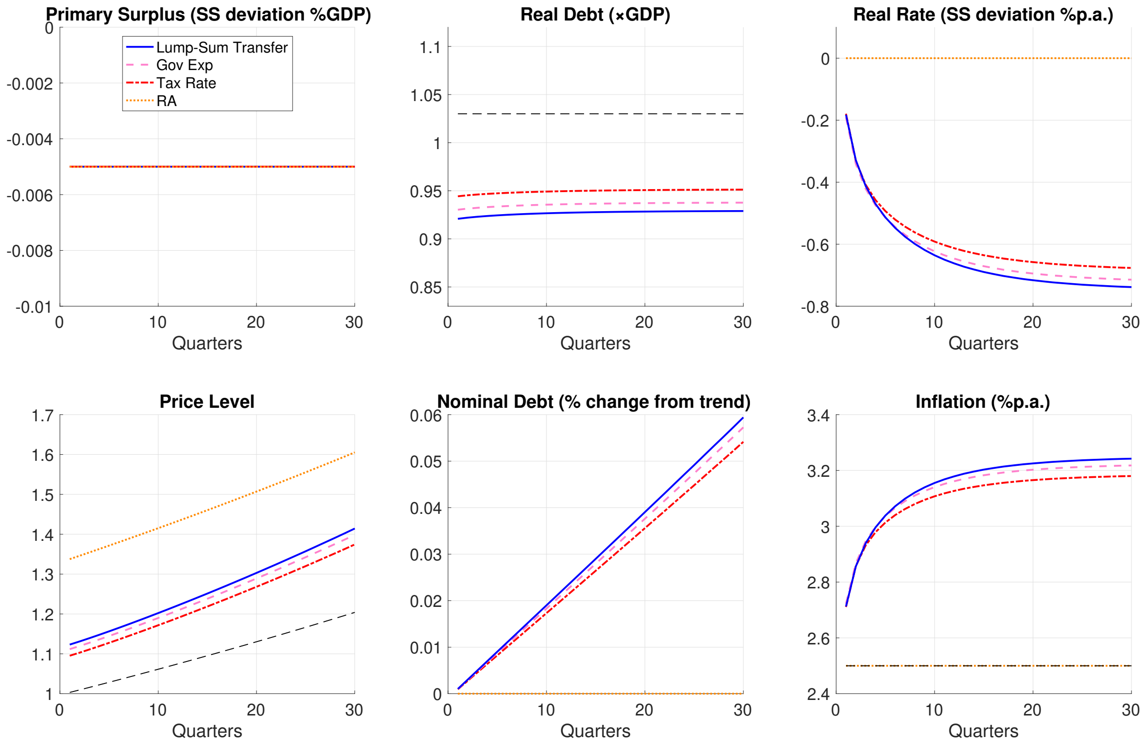
Why does an identical expansion in government debt place more upward pressure on the price level in the heterogeneous agent economy? The fiscal helicopter drop entails a redistribution of real wealth from high- to low-wealth households because the lump-sum transfer is progressive. Since the average MPC is higher among low wealth households, this redistribution raises the economy-wide desire to consume. With a constant aggregate endowment, the real interest rate must rise to restore goods market clearing. The higher (i.e. less negative) real interest payments require a reduction in total real government debt. Since nominal debt is fixed after the helicopter drop, the price level must then increase further. Equivalently, the additional spending pressure from redistribution, beyond the aggregate wealth effect, places more upward pressure on nominal prices than in the representative agent economy.

Decomposition of Fiscal Helicopter Drop. In addition to the direct redistributive impact of the fiscal helicopter drop, there are two additional indirect general equilibrium channels at play that shape the subsequent dynamics of the real rate and inflation. First, the upward jump in the price level redistributes wealth from savers to borrowers, and dilutes the real savings for households with a positive net nominal position. Second, the resulting rise in the real rate leads households to postpone consumption. The left panel in Figure 8 displays the dynamic effects of each of these channels on aggregate consumption. The helicopter drop itself raises consumption, while the higher price level lowers consumption. These effects diminish as the economy returns to steady-state. The higher real interest rate leads households to delay consumption, which is reflected by the initially lower (but subsequently higher) consumption in the green dotted line in left panel of Figure 8.

The aggregate decomposition masks substantial heterogeneity in the effect of these channels across households. The right panel of Figure 8 shows the contribution of each channel to the change in consumption on impact along the wealth distribution. Low-wealth households increase consumption substantially, predominantly due to their higher MPCs out of the direct helicopter drop at the steady-state price level. In addition, the jump in the price level induces households with negative wealth to increase their consumption because it lowers the real value of their debt. For households with positive wealth, the higher price level reduces their consumption because the real value of their nominal savings is curtailed. The higher real interest rate weakens consumption at impact for all households because of an intertemporal motive, except for households on the borrowing constraint. The dashed black line delineates the winners and losers of this experiment in terms of 2019 US dollars. Households with assets lower than $\$48,900$, which account for 55% of the population in our calibrated economy, gain from the helicopter drop.

Targeted vs Untargeted Fiscal Helicopter Drop. Figure 7 also shows that the initial increase in the price level is even larger when the helicopter drop is targeted towards poorer households. Compared to the untargeted case, the real interest rate rises by 0.8 additional percentage points on impact and, as a result, the price level jumps by an additional 3 percentage points (to 22%). In both the untargeted and targeted cases, the fiscal helicopter drop has a permanent effect on the price level and nominal government debt, but the inflationary effects are temporary. The saddle-path dynamics imply that both the real interest rate and the inflation rate return to their initial levels. In these experiments, the different price level responses between the heterogeneous agent and representative agent economies are mostly in terms of

Figure 9



Note: Impulse response to a permanent expansion in primary deficits. The dotted orange line shows the effects of a reduction in surplus in the Representative Agent model. The blue line labelled "Lump Sum" illustrates the dynamics following an expansion of lump sum transfers. The dashed red line labelled "Tax Rate" plots dynamics following a tax cut. The orange line plots dynamics in the representative agent (RA) model. The dashed black line plots the pre-shock trend.

timing. The higher initial rise in prices in the heterogeneous agent economy is followed by lower inflation, and the long-run cumulative increase in the price level is approximately the same in the two economies.

5.4 Permanent Deficit Expansion

Figure 9 displays impulse responses to a permanent deficit expansion from 2.0% to 2.5% of GDP. We consider two alternative policies for achieving a higher level of deficits. The solid blue line labeled "Lump-Sum Transfer" keeps the tax rate the same and raises the lump-sum transfer. The dashed red line labeled "Tax Rate" reduces the proportional tax rate. As was shown in Figure 5b, a permanent increase in deficits shifts the steady-state government null-cline to the right. Starting from the high real rate, low inflation steady-state, the long-run impact of the deficit expansion is to permanently lower both the real rate and the real value of government debt. These effects can be seen in the top row of Figure 9. The reduction in the value of real debt is achieved through a jump in the price level. In addition, because monetary policy does not respond, the lower real-rate translates into a permanently higher inflation rate. To prevent the permanent increase in deficits from leading to permanently higher inflation, the central bank would need to track the fall in the real rate by decreasing its nominal rate target. Campos et al. (2024) articulate this point in a two-asset HANK economy.

5.5 Additional Quantitative Results

We undertake a number of additional quantitative results to illustrate the robustness of our mechanism. Not For Publication Appendix N.5 considers a purely redistributive shock which entails no new issuance of government debt or changes primary deficits; Not For Publication Appendix N.6 studies a permanent change in primary deficits in an economy where households make a labor-leisure choice with endogenous output; Not For Publication Appendix N.7, considers alternative surplus reaction rules that guarantee a unique steady-state equilibrium. These extensions demonstrate the qualitative forces relating heterogeneity and precautionary savings to prices and inflation extend to richer environments.

6 Conclusion

We extend the fiscal theory of the price level to a heterogeneous-agent incomplete-market economy with flexible prices. In contrast to its representative agent counterpart, this model can be used to study an environment in which the government runs persistent deficits and the real rate is below the aggregate growth rate of the economy. This configuration is a more accurate representation of the current state of affairs in many developed economies.

Because our study is a first exploration in understanding how redistribution and precautionary savings can affect price level determinacy and inflation dynamics in this class of economies, we have intentionally focused on the limiting case of flexible prices. In reality, the price level does not jump instantaneously but moves more sluggishly. Rather, the initial bursts of inflation from these shocks are drawn out over a period of time. Despite this simplification, the general forces at work are informative about the two-way feedback between the equilibrium wealth distribution and movements in the price level.

In sticky-price economies with a particular form of heterogeneity that admits aggregation, Angeletos et al. (2023b, 2024) have shown analytically that demand-driven output movements can affect cumulative inflation via an expansion of the tax base, and have established a useful parallel between the inflationary impact of government spending in New Keynesian and FTPL models. Moreover, Kwicklis (2025) has shown that, in a calibrated HANK model, transfers to low-wealth households give rise to similar cumulative inflation but greater increases in real GDP than transfers to high-wealth ones. Further exploration of how fiscal policy interacts with nominal rigidities in the presence of heterogeneity and incomplete markets appears to be a promising avenue for future research. In particular, extending our model to a two-asset economy with both lower return nominal governments bonds and higher return productive assets as in Kaplan et al. (2018) would open the door to a quantitative framework with a richer characterization of the possible assets through which households can save.

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Online Appendix

A Representative Agent Economy

A.1 RA: Environment

Notation closely follows that of the main text. There exists a representative household that chooses real consumption flows $\tilde{c}_{jt}$ to maximize

$$\int e^{-\tilde{\rho}t} \frac{\tilde{c}_{jt}^{1-\gamma}}{1-\gamma} \mathrm{d}t \tag{A1}$$

Initial nominal assets A_0 are given. The household faces a flow budget constraint

$$dA_t = [i_t A_t + (1-\tau_t) P_t y_t - P_t \tilde{c}_t] dt \tag{A2}$$

subject to the borrowing constraint $A_t \geq 0$, where τ_t is a path of taxes set by the government. We may express the budget constraint in real de-trended terms as

$$\mathrm{d}a_t = [r_t a_t + (1-\tau_t) - c_t] \mathrm{d}t \tag{A3}$$

where the real rate is defined as $r_t := i_t - \pi_t - g$. Government debt dynamics follow

$$\mathrm{d}b_t = [r_t b_t - s_t] \mathrm{d}t \tag{A4}$$

where $s_t = \tau_t - x_t$.

We also impose two commonly maintained assumption in the fiscal theory of the price level: (i) the government can borrow, but not lend: $b_t \geq 0$; (ii) the price level P_t is differentiable for all $t > 0$. In this closed economy (i) is equivalent to $A_t \geq 0$.

Household Optimality. It is easy to show that the solution to the representative household problem yields the Euler equation

$$\frac{dc_t(a_t)}{c_t(a_t)} = \frac{1}{\gamma}(r_t - \rho) dt \tag{A5}$$

together with the household's transversality condition

$$\lim_{t\to\infty} e^{-\rho t} c_t^{-\gamma} a_t \leq 0 \tag{A6}$$

Monetary Policy. We allow for arbitrary monetary policy rules i_t, but assume that they lead to well-defined paths for inflation given real rates r_t (see Section 2.3).

A.2 RA: Equilibrium Definition

We now define a real equilibrium for this economy.

Definition 2. *Given τ_t and x_t, a real equilibrium is a collection of variables $\{c_t, a_t, b_t, r_t\}_{t\geq 0}$ such that:*

1. *For all $t > 0$, c_t satisfies the Euler equation (A5) and transversality condition (A6).*
2. *For all $t > 0$, a_t evolves according to the budget constraint (A3).*
3. *For all $t > 0$, b_t evolves according to the government budget constraint (A4).*
4. *For all $t \geq 0$, markets clear: $a_t = b_t$.*

Note that by Walras' Law, $c_t = 1 - x_t$ for all $t \geq 0$ so that the goods market clears.

A.3 RA: Uniqueness With Constant Surpluses

Next, we show that a real unique equilibrium exists whenever $\tau = \tau^* > 0$ and $x_t = x^*$, so that the government is running constant surpluses $s^* > 0$. First, note that the Euler equation (A5) along with market clearing for output $c_t = 1 - x^*$ implies that $r_t = \rho$ for all $t \geq 0$. Integrating the government budget constraint forwards then yields (A4):

$$b_0 = \lim_{T\to\infty}\left[\int_0^T e^{-\rho t}s^*\mathrm{d}t + e^{-\rho T}b_T\right] \tag{A7}$$

By transversality (A6) and market clearing, the latter term must be non-positive. Moreover, it cannot be negative as this would violate the non-negativity constraint on household assets and/or the assumption that the government cannot be a lender. Hence, it must be zero. But this then implies that

$$b_0 = \lim_{T\to\infty}\left[\int_0^T e^{-\rho t}s^*\mathrm{d}t\right] = \frac{s^*}{\rho} \tag{A8}$$

so b_0 is well-defined and strictly positive for any level of initial nominal assets B_0. The dynamics for real debt $\{b_t\}_{t>0}$ are then pinned down by the government budget constraint (A4). This proves the existence of a unique real equilibrium.

Given an initial level of nominal debt B_0, uniqueness of the real equilibrium implies uniqueness of the initial price level P_0. Subsequent inflation is uniquely pinned down by $r_t = \rho$, and a monetary policy rule which sets the path for the nominal rate i_t.

The Case of Deficits. The analysis above requires that the present discounted value in (A8) be finite and positive. Hence, running persistent deficits cannot be an admissible equilibrium under the assumption that (i) households face borrowing constraints or (ii) that aggregate government debt must be non-negative.

B Representative Agent with Bonds-In-Utility

B.1 Proof of Proposition 1

Proof. Our proof proceeds in several steps.

Step 1: Steady-state Uniqueness. Using (22) and $\mathbf{b}(r) = s^*/r$, the steady-state of the economy satisfies

$$r^* = \rho - \xi' \left(\frac{s^*}{r^*} \right) \tag{B1}$$

The LHS is increasing in r^*. The derivative of the RHS is given by:

$$\xi'' \left(\frac{s^*}{r^*} \right) \frac{s^*}{(r^*)^2} < 0 \tag{B2}$$

This is strictly negative when $r^* > 0$ under the assumed concavity of ξ. Moreover, if $s^* > 0$, any steady-state that features $b^* \geq 0$ must have $r^* > 0$. Hence, if a steady-state exists, it must be unique.

Step 2: Steady-state Existence. We first describe the limiting behavior of $\xi(\cdot)$. Note that $\xi'(a)$ is a strictly decreasing, positive function on $(0, \infty)$, so the limit $\ell \equiv \lim_{a\to\infty} \xi(a)$ exists. Suppose this limit is strictly positive. We have that:

$$\xi(a) = \xi(0) + \int_0^a \xi'(\tilde{a}) \mathrm{d}\tilde{a} \tag{B3}$$

$$\xi(a) > \xi(0) + a\ell \tag{B4}$$

$$\frac{\xi(a)}{a} > \frac{\xi(0)}{a} + \ell \tag{B5}$$

But taking limits of this expression contradicts the asymptotic boundedness condition of $\xi(\cdot)$. Hence, we must have that $\ell = 0$. Define:

$$I(r^*) \equiv \rho - \xi'\left(\frac{s^*}{r^*}\right) - r^* \tag{B6}$$

It follows that $\lim_{r^* \to 0+} I(r^*) > 0$ and that $\lim_{r^* \to \infty} I(r^*) \to \infty$. A unique solution to (B1) thus exists by the intermediate value theorem.

Step 3: Monotonicity of Real Assets. Note that the accumulation equation for assets is given by:

$$\dot{a}_t = (\rho - \xi'(a_t))\, a_t - s^* \tag{B7}$$

This can be written as

$$\frac{\dot{a}_t}{a_t} = \rho - \xi'(a_t) - \frac{s^*}{a_t} \tag{B8}$$

It follows from Step 1 that $\dot{a}_t > 0$ for $a_t > a^*$ and $\dot{a}_t < 0$ for $a_t < a^*$.

Step 4: Ruling Out Downwards Explosions. Next, we rule out all equilibria in which $a_{t'} < a^*$ for some $t' \geq 0$. By Step 3, $a_{t'} < a^*$ implies that $a_{t'} < a_t$ for all $t' \geq t$. Moreover, there are no limit points such that $\lim_{t \to \infty} a_t = a^{**}$ for any $a^{**} > 0$. Hence, any path in which $a_{t'} < a^*$ implies that the constraint $a_t \geq 0$ must be violated in finite time.

Step 5: Ruling Out Upwards Explosions. We now rule out equilibria in which $a_0 > a^*$. Integrating (B8), we obtain:

$$\log a_t - \rho t - \log a_0 = -\int_0^t \left(\xi'(a_u) + \frac{s^*}{a_u}\right) \mathrm{d}u \tag{B9}$$

Which yields

$$e^{-\rho t} a_t = a_0 \exp\{-K(t)\}, \quad \text{where} \quad K(t) \equiv \int_0^t \left(\xi'(a_u) + \frac{s^*}{a_u}\right) \mathrm{d}u \tag{B10}$$

We must show that $K(t)$ converges to a finite limit. To do this, observe that we may bound the growth rate of assets from below. In particular, we know that $\xi(a) \to 0$ as

$a \to \infty$. So we may pick a constant $\epsilon \in (0, \rho/2)$ such that

$$\dot{a}_t \geq (\rho - \epsilon)a_t - s^* \tag{B11}$$

Since a_t is increasing, we may pick a t' such that $s < \frac{1}{2}(\rho - \epsilon)a_{t'}$ for all $t > t'$. This implies that

$$\dot{a}_t \geq \frac{1}{2}(\rho - \epsilon)a_t \tag{B12}$$

for all $t > t'$. It follows that

$$\int_0^t \frac{s^*}{a_u}\mathrm{d}u < \int_0^t \frac{s^*}{a(t')\exp\{\frac{1}{2}(\rho - \epsilon)u\}}\mathrm{d}u < \infty \tag{B13}$$

Furthermore, observe that the concavity of $\xi(\cdot)$ implies that:

$$\xi'(a) \leq \frac{\xi(a) - \xi(0)}{a} \leq \frac{|\xi(a)| + \xi(0)}{a} \tag{B14}$$

where the last expression follows from the triangle inequality. Using the asymptotic boundedness condition, we have that:

$$\xi(a) \leq Ca^{\varepsilon - 1} \tag{B15}$$

for some $C > 0$ and for all $a \geq a'$, where a' is an arbitrarily large but finite threshold. Hence, there exists a $t' > 0$ such that

$$\int_{t''}^t \xi'(a_u)\mathrm{d}u < \int_{t''}^t Ca_u^{\varepsilon - 1}\mathrm{d}u \tag{B16}$$

Since $\varepsilon \in (0, 1)$ and a_t grows at a rate faster than $\frac{1}{2}(\rho - \varepsilon)$ asymptotically, we have that

$$\int_{\hat{t}}^t \xi'(a_u)\mathrm{d}u < \int_{\hat{t}}^t Ca(\hat{t})^{\varepsilon - 1}\exp\left\{\frac{1}{2}(\varepsilon - 1)(\rho - \epsilon)u\right\}\mathrm{d}u \tag{B17}$$

where $\hat{t} = \max\{t', t''\}$. It follows that $\lim_{t \to \infty} K(t) < \infty$. But (B10) then implies that $e^{-\rho t}a_t > 0$, which is a violation of the household's transversality condition. Hence, $a_0 > a^*$ cannot be an equilibrium. It follows that $a_t = a_0 = a^*$ is the only real equilibrium of this economy. $\square$

B.2 Proof of Proposition 2

Proof. If there are no steady-states, then a_t is a function that is either increasing or decreasing with no limit points. From Proposition 1, we know that decreasing paths of a_t violate the households' borrowing constraint, while increasing paths violate household optimality. This proves the first statement.

Suppose now that that two steady-states exists. It suffices to prove that the steady-state is locally unstable. Observe that the stability properties of the steady-state are given by the sign of:

$$\dot{a}_t = (\rho - \xi'(a_t))a_t - s^* \tag{B18}$$

$$= \frac{1}{a_t}\left[\rho - \xi'(a_t) - \frac{s^*}{a_t}\right] \tag{B19}$$

At the top steady-state a_H^*, it must be that $\rho - \xi'(a) > s^*/a$ for all $a \in (a_H^*, a_H^* + \varepsilon)$ for some $\varepsilon > 0$. Suppose otherwise. The above expression cannot hold with equality for any other $a \in (a_H^*, a_H^*, \varepsilon)$ since that would contradict a_H^* being the top steady-state. The only remaining possibility is that $\rho - \xi'(a) < s^*/a$. Furthermore, we know that $\xi'(a) \to 0$ as $a \to \infty$, so $\rho - \xi'(a)$ must eventually exceed s^*/a, which is negative. Since there are no other limit points, any initial $a_0 > a_H^*$ will make debt grow at an exponential rate. From the proof of Proposition 1, this violates the household's transversality condition. Hence, $a_0 \in (a_H^*, \infty)$ are not admissible equilibria.

Next, observe that at the bottom steady-state a_L^*, we must have that $\rho - \xi'(a) < s^*/a$ for any $a \in (a_L^* - \varepsilon, a_L^*)$. As before, the above expression cannot hold with equality for any other a in this left-neighborhood. Suppose now that the inequality is reversed. In this case, we have that $\rho - \xi'(a)$ remains finite as $a \to 0$, while $s^*/a \to -\infty$. By the intermediate value theorem, there must exist another $a \in (0, a_L^*)$ in which $\dot{a}_t = 0$, thus contradicting a_L^* being the bottom steady-state. It follows that all $a_0 \in (0, a_L^*]$ are admissible equilibria because all paths of debt from these initial conditions will eventually converge to a_L^*. Moreover, all paths of debt in $a_0 \in (a_L^*, a_H^*]$ must eventually converge to either the bottom or top steady-state, since the sign of $\dot{a}_t$ remains constant within this range. □

B.3 Uniqueness with Zero Surpluses

The government accumulation equation with zero surpluses is

$$\mathrm{d}b_t = [r(b_t)b_t]\mathrm{d}t \tag{B20}$$

with $r(b_t) = \rho - \xi'(b_t)$ This implies a steady-state interest rate of $r^* = 0$ whenever $\mathbf{a}(0) > 0$, with an associated steady-state level of real debt given by $b^* = \mathbf{a}(0)$. The first-order dynamics of this system around the steady-state are given by:

$$\mathrm{d}b_t = [b^* r'(b_t)]\mathrm{d}t \tag{B21}$$

The right-hand side is strictly positive given that $\xi'(\cdot)$ is strictly increasing. Hence, the steady-state is unstable. Since $B_0 > 0$ is given, there exists a unique, finite value of P_0 such that the equilibrium converges back to the steady-state. There are also a continuum of stationary real equilibria with $P = \infty$, in which $r < \underline{r}$ and aggregate real debt is zero.

C Additional Details on Long-Run Anchoring

In this section, we demonstrate how the monetary authority can eliminate all dynamic equilibria that converge to the high inflation steady-state, leaving only a unique equilibrium that leads to the saddle-path stable, low-inflation steady-state. Concretely, suppose the monetary authority has the power to coordinate private sector beliefs about long-run inflation. Under such a setting we envisage two pillars of central bank policy: (i) a path or rule for short-term nominal interest rates i_t, and (ii) a long-run inflation target π^*. Whereas the interest rate is a policy tool that the central bank directly implements by intervening in appropriate markets or paying interest on reserves, the long-run inflation target is no more than an attempt to coordinate beliefs. If:

(i) the long-run inflation target and the long-run nominal interest rate (π^*, i^*) are set to be consistent with the equilibrium real rate at the saddle-path steady-state, $i^* - \pi^* - g = r^*_H$;

(ii) fiscal policy follows a constant deficit policy or a passive interest payment reaction rule with $\phi_s < 1$, so that the high real rate, low inflation steady-state is saddle-path stable;

(iii) private sector beliefs about long-run inflation are consistent with the central bank's target, then there is a unique real equilibrium and the price level and inflation are pinned down for all t. The third of these conditions is a big "if", and there is no fundamental reason to expect it to hold. However the key point is that managing *long-run* inflation expectations is sufficient to pin down the price level and inflation in the *short-run*. If the central bank is successful at convincing the private sector to coordinate on a long-run inflation target, then this is sufficient to eliminate any indeterminacy about inflation at all points in time. Note that anchoring long-run inflation expectations at π^* does not assume away the issue of price level determination in the short-run. Both the initial price level and subsequent inflation remain endogenous and depend on monetary policy, fiscal policy and private sector behavior.

Even with long-run inflation anchoring, fiscal policy remains an essential component of price level determination. Coordinating long-run expectations only uniquely determines the price level in the short-run if fiscal policy acts in a way that ensures the saddle-path stability of the low-inflation steady-state. Such fiscal policy settings are the same as those required for uniqueness in the case with persistent surpluses.

D Additional Fiscal Rules

D.1 Real Rate Reaction Rule

Suppose the government follows a fiscal rule of the form:

$$s_t = s^* + \phi_r(r_t - r^*) \tag{D1}$$

where $s^* = r^* b^*$ is consistent with any given point on the household demand curve, so that the tuple $(b^*, r^*) = (\mathbf{a}(r^*), r^*)$ with $r^* < 0$. The government accumulation equation is:

$$\mathrm{d}b_t = [r_t b_t - s_t]\mathrm{d}t \tag{D2}$$

The nullclines of the government accumulation equation are then defined by the following function:

$$r(b) = \frac{(b^* - \phi_r)r^*}{b - \phi_r} \tag{D3}$$

Relative to a fixed surplus rule, the real-rate shifts the government nullcline upwards, as in Figure 3.

The slope of this function is given by

$$\frac{\mathrm{d}r}{\mathrm{d}b} = -\frac{(b^* - \phi_r)r^*}{(b - \phi_r)^2} \tag{D4}$$

which is strictly positive whenever $b^* > \phi_r$. Moreover, the nullcline intersects the r-axis at a negative real interest rate that is greater than $\underline{r}$ if

$$\phi_r < \frac{s^*}{r^* - \underline{r}} \tag{D5}$$

Local Determinacy. We now examine conditions for this fiscal rule to yield local determinacy. The equilibrium dynamics are given by:

$$\mathrm{d}b_t = [r(b_t)(b_t - \phi_r) - (r^* - \phi_r)b^*]\mathrm{d}t \tag{D6}$$

where $r(b_t) = \rho - \xi'(b_t)$. The first-order dynamics of this system around the steady-state are given by:

$$\mathrm{d}b_t = [r(b^*) + (b^* - \phi_r)r'(b^*)](b_t - b^*)dt \tag{D7}$$

Under the assumption that the real-rate rule implements a unique steady-state, we must have that:

$$r'(b^*) > -\frac{r^*}{b^*} \tag{D8}$$

where the right hand side is the slope of the government nullcline since

$$\frac{\mathrm{d}}{\mathrm{d}b}\frac{s^*}{b^*} = -\frac{s^*}{(b^*)^2} = -\frac{r^*}{b^*} \tag{D9}$$

Given a unique steady-state, the household asset demand curve must intersect the nullcline of government debt from below. Hence, a sufficient condition for the right-hand side of (D7) to be positive is $\phi_r < 0$.

Global Determinacy. Following the steps in the proof of Proposition 1, the dynamics of debt can be written as

$$\log b_t - \rho t - \log b_0 = -\int_0^t \left(\xi'(b_u) - \phi_r \frac{\rho - \xi'(b_u)}{b_u} + \frac{r^* - s^*}{b_u}\right) \mathrm{d}u \tag{D10}$$

Since (b^*, r^*) is the unique steady-state that is locally unstable, any $b_0 < b^*$ will give rise to trajectories in which debt is monotonically decreasing, while $b_0 > b^*$ will give rise to debt that is monotonically increasing. Downward trajectories will violate the household borrowing constraint in finite time. From the proof of Proposition 1, we know that the right-hand side of equation (D10) is bounded. Hence, upward trajectories violate the household's transversality condition. This proves that the only admissible equilibrium is one in which $b_0 = b^*$.

D.2 Real Debt Reaction Rule

Our argument for uniqueness proceeds in three steps. First, we derive conditions for a unique steady-state. Second, we derive conditions for the steady-state to be saddle-path stable. This ensures local uniqueness. Finally, we consider whether explosive paths in debt can be ruled out globally. This ensures global uniqueness.

Steady-State Uniqueness. Suppose the government follows a fiscal rule of the form:

$$s_t = s^* + \phi_b(b_t - b^*) \tag{D11}$$

where $s^* = r^* b^*$ is consistent with any given point on the household demand curve, so that the tuple $(b^*, r^*) = (\mathbf{a}(r^*), r^*)$ with $r^* < 0$. The government accumulation equation is:

$$\mathrm{d}b_t = [r_t b_t - s_t]\mathrm{d}t \tag{D12}$$

The nullclines of the government accumulation equation are then defined by the following function:

$$r(b) = \frac{s^* - \phi_b b^*}{b} + \phi_b \tag{D13}$$

A sufficient condition for steady-state uniqueness is that this function is downwards sloping. This will ensure that it intersects the upwards sloping steady-state demand curve $\mathbf{a}(r)$ exactly once. The slope of this function is

$$\frac{\mathrm{d}r}{\mathrm{d}b} = -\frac{s^* - \phi_b b^*}{b^2} \tag{D14}$$

$$= -\frac{r^* b^* - \phi_b b^*}{b^2} \tag{D15}$$

which is (weakly) negative whenever $r^* \geq \phi_b$. Hence, $\phi_b \leq r^* < 0$ is sufficient for steady-state uniqueness.

Local Determinacy. We now examine conditions for this fiscal rule to give rise to local determinacy. The equilibrium dynamics are given by:

$$\mathrm{d}b_t = [r(b_t)(b_t - \phi_r) - (r^* - \phi_r)b^*]\mathrm{d}t \tag{D16}$$

where $r(b_t) = \rho - \xi'(b_t)$. The first-order dynamics of this system around the steady-state are given by:

$$db_t = [r(b^*) - \phi_b + b^* r'(b^*)](b_t - b^*)dt \tag{D17}$$

The last term is positive because $\xi(\cdot)$ is upward sloping by assumption. The sum of the first two terms are positive under the assumption $r^* > \phi_b$. This proves local determinacy.

Global Determinacy. Following the steps in the proof of Proposition 1, the dynamics of debt can be written as

$$\log b_t - \rho t - \log b_0 = -\int_0^t \left(\xi'(b_u) - \phi_r \frac{\rho - \xi'(b_u)}{b_u} + \frac{r^* - s^*}{b_u} \right) \mathrm{d}u \tag{D18}$$

Since (b^*, r^*) is the unique steady-state that is locally unstable, any $b_0 < b^*$ will give rise to trajectories in which debt is monotonically decreasing, while $b_0 > b^*$ will give rise to debt that is monotonically increasing. Downward trajectories will violate the household borrowing constraint in finite time. From the proof of Proposition 1, we know that the right-hand side of equation (D18) is bounded. Hence, upward trajectories violate the household's transversality condition. This proves that the only admissible equilibrium is one in which $b_0 = b^*$.

D.3 Interest Payment Reaction Rule

Steady-State Invariance. Suppose the government follows the fiscal rule:

$$s_t = s^* + \phi_s(r_t b_t - r^* b^*) \tag{D19}$$

where $s^* = r^* b^*$ is consistent with any given point on the household demand curve, so that the tuple $(b^*, r^*) = (\mathbf{a}(r^*), r^*)$ with $r^* < 0$. The government accumulation equation is:

$$db_t = [r_t b_t - s_t]dt \tag{D20}$$

The null-clines of the government accumulation equation are then defined by the following function:

$$r(b) = \frac{s^* - \phi_s r^* b^*}{b - \phi_s b} = \frac{s^*}{b} \tag{D21}$$

which shows that the steady-states are unchanged. Hence, there is no scope for this fiscal rule to eliminate steady-state multiplicity.

Local Dynamics. The dynamics of government debt are given by

$$db_t = (1 - \phi_s)\left(r(b_t)b_t - s^*\right)dt \tag{D22}$$

It follows that the stability properties of the two steady-states in the baseline case with $\phi_s = 0$ are reversed when $\phi_s > 1$.

D.4 Nominal Debt Growth Targeting Rule

We now analyze a nominal debt growth targeting rule, as in Hagedorn (2021). For simplicity, we assume the growth rate of output is zero, $g = 0$. Suppose the government fixes the growth rate of nominal debt at a target g^B. Hence, the government debt accumulation equation must satisfy:

$$g^B = \frac{\dot{B}_t}{B_t} = i^* - P_t \frac{s_t}{B_t} \tag{D23}$$

In steady-state, we must have that:

$$s^* = (i^* - g^B)b^* \tag{D24}$$

Since real debt is constant, the growth rate of nominal debt is also the steady-state inflation rate. Hence, for a given tuple (b^*, r^*) consistent with household asset demand, the government chooses s^* such that

$$s^* = r^* b^* \tag{D25}$$

and g^B such that $g^B = i^* - r^*$. It follows that the government can change g^B to target different steady-states.

Observe that we can rearrange Equation (D23) to write the implied surplus rule s_t as:

$$s_t = (i^* - g^B)b_t = r^* b_t \tag{D26}$$

Therefore, this rule is isomorphic to a real debt reaction rule with coefficient r^*. It follows from Section D.2 that this rule gives rise to steady-state uniqueness and global determinacy.

E Additional Derivations for the HA Economy

E.1 Existence of $\underline{r}$

This subsection shows that there exists a finite $\underline{r}$ such that no household saves in a stationary equilibrium if $r \leq \underline{r}$. Suppose no such $\underline{r}$ exists. Then, there must exist a non-zero mass of households who are unconstrained in any stationary equilibrium, for all $r < \rho$.

If a household is unconstrained, their marginal utility must satisfy the Euler Equation (see the derivation of equation E9):

$$\begin{aligned}(\rho - r_t)u'(c_t(a,z)) = &\sum_{z' \neq z} \lambda_{zz'}[u'(c_t(a,z')) - u'(c_t(a,z))] \\ &+ u''(c_t(a,z))[\partial_t c_t(a,z) + \varsigma_t(a,z)\partial_a c_t(a,z)]\end{aligned} \tag{E1}$$

where recall that $\varsigma(a,z)$ denotes the household's savings function. Note that marginal utility is bounded from below since $\min \mathcal{Z} > 0$ and $\partial_a c_t(a,z) > 0$. Moreover, $\partial_t c_t(a,z) = 0$ in a stationary equilibrium. We also have that $(\rho - r_t)u'(c_t(a,z)) \to \infty$ as $r_t \to -\infty$. Hence, there must exist a $r_t \leq \underline{r}$ for some $\underline{r} \in \mathbb{R}$ such that the savings rate of all households is negative for all $a \in \mathbb{R}_{++}$ and $z \in \mathcal{Z}$. But this violates the assumption that there exists a strictly positive mass of households that are unconstrained in stationary equilibrium.

E.2 Steady-State Welfare Comparison

We show that steady-states with higher real interest rates are Pareto ranked for any initial condition of assets a_{jt} and income z_{jt}. In particular, consider a particular profile of income shocks $\{z_{jt}\}_{t\geq 0}$ that induces a (realized) consumption and savings streams $\{c_{jt}, a_{jt}\}$ under a constant real interest rate r_L^*. This consumption plan can also be implemented at a higher interest rate $r_H^* > r_L^*$ for the same sequence of income shocks, since the change in savings in any given period will be:

$$\mathrm{d}a_{jt} = [(r_H^* - r_L^*)a_{jt}]\mathrm{d}t \tag{E2}$$

which is weakly positive for any given $a_{jt} > 0$ (recall that the surplus s^*, and hence taxes and transfers, are fixed and independent of the level of the real interest rate). Higher interest rates weakly expand the budget set of all households for any given a_{j0} and z_{j0}. This proves that a steady-state with r_H^* Pareto dominates r_L^*.[25]

E.3 Derivation of Optimal Consumption Dynamics

This section derives expressions for the consumption dynamics of unconstrained and constrained households.

Unconstrained Households. We show that the expected consumption dynamics for unconstrained households are given by

$$\frac{\mathbb{E}_t\left[\mathrm{d}c_{jt}\right]}{c_{jt}} = \frac{1}{\gamma}\left(r_t - \rho\right)\mathrm{d}t + \frac{1}{\gamma}\sum_{z'}\lambda_{z_{jt}z'}\left(\frac{c_t(a_j, z')}{c_{jt}}\right)^{-\gamma}\mathrm{d}t + \sum_{z'}\lambda_{z_{jt}z'}\left(\frac{c_t(a_j, z')}{c_{jt}}\right)\mathrm{d}t \tag{E3}$$

Here we use the short-hand notation $c_{jt} := c_t(a_j, z_j)$ to denote the consumption of household j at time t. Recall the HJB Equation:

$$\rho v_t(a, z) = \max_c u(c) + \varsigma_t(a, z)\partial_a v(a, z) + \sum_{z'\neq z}\lambda_{z,z'}[v_t(a, z') - v_t(a, z)] + \partial_t v_t(a, z) \tag{E4}$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\varsigma_t(a, z)$ is the savings function (10). The FOC is:

$$u'(c) = \partial_a v_t(a, z) \tag{E5}$$

[25] This proof strategy follows Aguiar et al. (2021), who construct robust Pareto-improving policies in the presence of capital accumulation.

Differentiating the above with respect to a yields

$$u''(c_t(a,z))\partial_a c_t(a,z) = \partial^2_{aa} v_t(a,z) \tag{E6}$$

Differentiating with respect to t yields

$$u''(c_t(a,z))\partial_t c_t(a,z) = \partial^2_{at} v_t(a,z) \tag{E7}$$

The envelope condition for (E4) is:

$$\rho\partial_a v_t(a,z) = \partial^2_{aa} v_t(a,z)\varsigma_t(a,z) + r_t\partial_a v_t(a,z) + \sum_{z'\neq z}[\partial_a v_t(a,z') - \partial_a v_t(a,z)] + \partial^2_{at} v_t(a,z) \tag{E8}$$

Using (E6) and (E7) into the equation above yields:

$$\begin{aligned}(\rho - r_t)u'(c_t(a,z)) = &\sum_{z'\neq z}\lambda_{zz'}[u'(c_t(a,z')) - u'(c_t(a,z))] \\ &+ u''(c_t(a,z))[\partial_t c_t(a,z) + \varsigma_t(a,z)\partial_a c_t(a,z)]\end{aligned} \tag{E9}$$

(E9) holds at any point on the interior of the state space $a > 0$ (i.e. for all unconstrained households). Using Ito's lemma for jump processes, we can write it as:

$$(\rho - r_t)u'(c_t(a_j,z_j)) = \frac{\mathrm{d}\mathbb{E}[u'(c_t(a_j,z_j))]}{\mathrm{d}t} \tag{E10}$$

Furthermore, using Ito's lemma on $c_t(a_j,z_j)$ yields

$$\begin{aligned}\mathrm{d}c_j = \Bigg[&\partial_a c_t(a_j,z_j)\varsigma_t(a_j,z_j) + \partial_t c_t(a_j,z_j) \\ &+ \sum_{z'\neq z_j}\lambda_{z_j z'}[c_t(a_j,z') - c_t(a_j,z_j)]\Bigg]\mathrm{d}t + [c_t(a_j,z') - c_t(a_j,z_j)]\mathrm{d}\tilde{N}_j\end{aligned} \tag{E11}$$

where $\tilde{N}_j$ is the compensated Poisson process for the stochastic process of income z' (*i.e.*, a Poisson process made into a martingale by subtracting its mean). Expected consumption therefore follows:

$$\mathbb{E}[\mathrm{d}c_j] = \left[\partial_a c_t(a_j,z_j)\varsigma_t(a_j,z_j) + \partial_t c_t(a_j,z_j) + \sum_{z'\neq z_j}\lambda_{z_j z'}[c_t(a_j,z') - c_t(a_j,z_j)]\right]\mathrm{d}t \tag{E12}$$

We may combine this with (E9) to obtain

$$
\begin{aligned}
(\rho - r_t)u'(c_t(a_j, z_j)) &= \sum_{z' \neq z_j} \lambda_{z_j z'}[u'(c_t(a_j, z')) - u'(c_t(a, z_j))] \\
+u''(c_t(a_j, z_j))\frac{\mathbb{E}[\mathrm{d}c_j]}{\mathrm{d}t} &- u''(c_t(a_j, z_j)) \sum_{z' \neq z_j} \lambda_{z_j z'}[c_t(a_j, z') - c_t(a_j, z_j)]
\end{aligned}
\tag{E13}
$$

This yields (E3) after dividing by $u''(c_t(a_j, z_j))$ and specializing to $u'(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Constrained Households. We show that the expected consumption dynamics for borrowing constrained households satisfy:

$$
\frac{\mathbb{E}_t\left[\mathrm{d}c_{jt}\right]}{c_{jt}} = \sum_{z'} \lambda_{z_{jt}, z'} \left(\frac{c_t(a_j, z')}{c_{jt}}\right) \mathrm{d}t. \tag{E14}
$$

The consumption dynamics for constrained households are given by

$$
\mathrm{d}c_t(0, z_j) = \sum_{z' \neq z_j} \lambda_{z_j z'}[c_t(0, z') - c_t(0, z_j)]\mathrm{d}t + [c_t(0, z') - c_t(0, z_j)]\mathrm{d}\tilde{N}_j \tag{E15}
$$

since households consume their (post-tax) income whenever constrained (until receiving a more favorable income draw). Taking expectations and dividing by $c_t(0, z_j)$ then yields (E14) directly. Finally, we note that in the recursive formulation of equation (28) in the main text we used the notation $c(a_j, z_j, \Omega_t)$ in place of $c_t(a_j, z_j)$.

E.4 Derivation of Real Rate Functional

This section derives the real interest rate functional given in equation (28). We start from the characterization of optimal consumption dynamics contained in Appendix E.3. Namely, we use (E11) and (E15) to integrate across all households j:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} \int_j c_t(a_j, z_j)\mathrm{d}j = \int_{j:\mathrm{u}} \Bigg(&\partial_a c_t(a_j, z_j) \varsigma_t(a_j, z_j) \\
&+ \partial_t c_t(a_j, z_j) + \sum_{z' \neq z_j} \lambda_{z_j z'}[c_t(a_j, z') - c_t(a_j, z_j)] \Bigg) \mathrm{d}j \\
&+ \int_{j:\mathrm{c}} \sum_{z' \neq z_j} \lambda_{z_j z'}[c_t(0, z') - c_t(0, z_j)]
\end{aligned}
\tag{E16}
$$

where the $\mathrm{d}\tilde{N}_j$ terms vanish by the exact law of large numbers (Duffie and Sun, 2007, 2012). The first integral on the right-hand side is over unconstrained households (j : u), while the second integral is over constrained households (j : c). Note that the above equation must be equal to zero, since $\int_j c_t(a_j, z_j)\mathrm{d}j = 1 - x^*$, by market clearing. Dividing by $u''(c_t(a_j, z_j))$ in (E9) and using CRRA preferences, we obtain:

$$-\frac{1}{\gamma}(\rho - r_t)c_t(a_j, z_j) = \sum_{z' \neq z_j} \lambda_{zz'} \frac{1}{u''(c_t(a_j, z_j))} \left[u'(c_t(a_j, z')) - u'(c_t(a_j, z_j))\right] + \partial_t c_t(a_j, z_j) + \varsigma_t(a_j, z_j)\partial_a c_t(a_j, z_j) \tag{E17}$$

Integrating over all unconstrained agents and using (E16) to substitute for $\partial_t c_t(a_j, z_j) + s_t(a_j, z_j)\partial_a c_t(a_j, z_j)$ yields

$$\begin{aligned} -\frac{1}{\gamma}(\rho - r_t) \int_{j:\mathrm{u}} c_t(a_j, z_j)\mathrm{d}j = \\ \int_{j:\mathrm{u}} \sum_{z' \neq z_j} \lambda_{z_j z'} \frac{1}{u''(c_t(a_j, z_j))} \big[u'(c_t(a_j, z')) - u'(c_t(a_j, z_j))\big]\mathrm{d}j \\ - \int_j \sum_{z' \neq z_j} \lambda_{z_j z'}(c_t(a_j, z') - c_t(a_j, z_j))\mathrm{d}j \end{aligned} \tag{E18}$$

Using CRRA utility, and the fact that $\sum_{z' \neq z_j} \lambda_{z_j z'} = -\lambda_{z_j z_j}$, we may write the above expression as

$$r_t = \rho - \frac{\int_{j:u} c_t(a_j, z_j) \left[\sum_{z'} \lambda_{z_j z'} \left(\frac{c_t(a_j, z')}{c_t(a_j, z_j)}\right)^{-\gamma}\right] \mathrm{d}j + \gamma \int_j c_t(a_j, z_j) \left[\sum_{z'} \lambda_{z_j z'} \frac{c_t(a_j, z')}{c_t(a_j, z_j)}\right] \mathrm{d}j}{\int_{j:\mathrm{u}} c_t(a_j, z_j)\mathrm{d}j} \tag{E19}$$

Note that, by splitting the last integral in the numerator of (E19) into unconstrained and constrained agents, we may also write this equation as

$$\frac{\mathcal{C}_t^{\mathrm{u}}}{\gamma}(r_t - \rho) + \mathcal{C}_t^u \tilde{\mathbb{E}}_t^{\mathrm{u}} \left[\frac{1}{\gamma} \sum_{z'} \lambda_{z_j z'} \left(\frac{c_t(a_j, z')}{c_{jt}}\right)^{-\gamma} + \sum_{z'} \lambda_{z_j z'} \frac{c_t(a_j, z')}{c_{jt}}\right] + \mathcal{C}_t^{\mathrm{c}} \tilde{\mathbb{E}}_t^{\mathrm{c}} \left[\sum_{z'} \lambda_{z_j z'} \frac{c_t(a_j, z')}{c_{jt}}\right] = 0 \tag{E20}$$

where $\tilde{\mathbb{E}}_t^{\mathrm{u}}[\cdot] = \mathbb{E}_t[\frac{c_t(a_j,z_j)}{\mathbb{E}_t[c_t(a_j,z_j)|a_j>0]}(\cdot)|a_j > 0]$, $\tilde{\mathbb{E}}_t^{\mathrm{c}}[\cdot] = \mathbb{E}_t[\frac{c_t(a_j,z_j)}{\mathbb{E}_t[c_t(a_j,z_j)|a_j=0]}(\cdot)|a_j = 0]$ are the consumption-weighted averages of unconstrained and constrained households, respectively. $\mathcal{C}_t^{\mathrm{u}} = \int_{j:\mathrm{u}} c_t(a_j, z_j)\mathrm{d}j$ is the total consumption of unconstrained agents and $\mathcal{C}_t^{\mathrm{c}} = \int_{j:\mathrm{c}} c_t(a_j, z_j)\mathrm{d}j$ is the total consumption of all households.

The first term of (E20) represents the intertemporal substitution motive (γ^{-1} is the EIS), the second term represents the precautionary saving motive due to prudence

($\gamma+1$ is the coefficient of relative prudence), and the third term measures the severity of borrowing constraints. That the second term of (E20) captures the precautionary saving motive due to prudence can be verified by taking a second-order approximation of $c_t(a_j, z')^{-\gamma}$ around $c_t(a_j, z_j)$ which yields the more transparent expression

$$\frac{\mathcal{C}_t^{\mathrm{u}}}{\gamma}(r_t-\rho)+\frac{\gamma+1}{2}\mathcal{C}_t^{u}\tilde{\mathbb{E}}_t^{\mathrm{u}}\left[\sum_{z'}\lambda_{z_jz'}\left(\frac{c_t(a_j,z')}{c_{jt}}-1\right)^2\right]+\mathcal{C}_t^{\mathrm{c}}\tilde{\mathbb{E}}_t^{\mathrm{c}}\left[\sum_{z'}\lambda_{z_jz'}\left(\frac{c_t(a_j,z')}{c_{jt}}-1\right)\right]=0 \quad \text{(E21)}$$

Details of this approximation are in the Not for Publication Appendix N.1. The interest rate can be written as a functional in terms of aggregate states by replacing $c_t(a_j, z_j)$ with $c(\omega_j, z_j, \Omega_t)$. Equation (28) then follows directly from (E21).

E.5 Finite Difference Approximation

We begin by deriving the Kolmogorov Forward Equation (KFE) for wealth shares. Note that the dynamics for wealth shares $\omega_{jt} = \frac{a_{jt}}{a_t}$ is given by

$$\frac{\mathrm{d}\omega_{jt}}{\omega_{jt}\mathrm{d}t} = \frac{\mathrm{d}a_{jt}}{a_{jt}\mathrm{d}t} - \frac{\mathrm{d}b}{b_t\mathrm{d}t} \quad \text{(E22)}$$

Using Equations (6) and (18) yields

$$\frac{\mathrm{d}\omega_{jt}}{\mathrm{d}t} = \omega_{jt}\left(\frac{r_t a_{jt} + z_{jt} - \tau(z_{jt}) - c_{jt}}{a_{jt}} - \frac{r_t b_t - s^*}{b_t}\right) \quad \text{(E23)}$$

$$\frac{\mathrm{d}\omega_{jt}}{\mathrm{d}t} = \frac{z_{jt} - \tau(z_{jt}) - c_{jt} + \omega_{jt}s^*}{b_t} \quad \text{(E24)}$$

This implies that the KFE for wealth shares is given by:

$$\partial_t f_t(\omega, z) = \mathcal{A}_\omega^*[f_t, b_t](\omega, z) + \mathcal{A}_z^*[f_t](z) \quad \text{(E25)}$$

where

$$\mathcal{A}_\omega^*[f_t, b_t](\omega, z) = -\partial_\omega\left[f_t(\omega, z)\frac{z - \tau(z) - c(\omega, z, \Omega_t) + \omega s^*}{b_t}\right] \quad \text{(E26)}$$

and

$$\mathcal{A}_z^*[f_t](z) = -f_t(\omega, z)\sum_{z'\neq z}\lambda_{zz'} + \sum_{z'\neq z}\lambda_{z'z}f_t(\omega, z') \quad \text{(E27)}$$

where we have made the dependence of the consumption function on aggregate state variables explicit. Note further that these operators are adjoint to underlying operators $\mathcal{A}_\omega$ and $\mathcal{A}_z$.

We may discretize the distribution $f(\omega, z)$ into $N = N_\omega \times N_z$ discrete points, where N_ω is a discrete grid for ω of width Δ_ω. We denote the discretized distribution as $\mathfrak{f}$ and write the dynamics of the joint system as

$$\frac{\mathrm{d}\mathfrak{f}_t}{\mathrm{d}t} = \mathbf{A}_\omega\left[\mathfrak{f}_t, b_t\right]^T \mathfrak{f}_t + \mathbf{A}_z^T \mathfrak{f}_t \tag{E28}$$

$$\frac{\mathrm{d}b_t}{\mathrm{d}t} = \mathbf{r}\left[\mathfrak{f}_t, b_t\right] b_t - s^* \tag{E29}$$

The interest rate functional $\mathbf{r}\left[\mathfrak{f}_t, b_t\right]$ corresponds to the interest rate functional in Equation (28) where we have substituted for the discretized endowment share distribution. The matrix $\mathbf{A}_\omega\left[\mathfrak{f}_t, b_t\right]^T$ is a finite difference approximation to $\mathcal{A}_\omega[f_t, b_t]$ using the appropriate upwind scheme (Achdou et al., 2022). Hence, it is a tridiagonal matrix which consists of the following terms:

$$\left\{0, -\frac{z - \tau(z) - c(\omega, z, \Omega_t) + \omega s^*}{b_t \Delta_\omega}, \frac{z - \tau_t(z) - c(\omega, z, \Omega_t) + \omega s^*}{b_t \Delta_\omega}\right\} \tag{E30}$$

The matrix $\mathbf{A}_z$ is the Markov transition matrix for z in the product space $\omega \times z$. Note that this matrix does not depend on $\mathfrak{f}_t$ or b_t. The rows of both $\mathbf{A}_\omega\left[\mathfrak{f}_t, b_t\right]$ and $\mathbf{A}_z$ sum to zero to ensure that $\mathfrak{f}_t$ preserves mass. The linearized system can then be exactly expressed as (34) if the effect of $\mathfrak{f}$ on the interest rate is small. For instance, Equation (34) holds exactly when consumption functions are linear in wealth, because the interest rate is invariant to changes in wealth shares. However, because the interest rate functional uses a consumption-based aggregator, in practice it is only necessary for the consumption function to be linear amongst high-wealth households, who consume relatively more of the aggregate endowment.

E.6 Ruling Out Explosive Equilibria

Downward explosion paths are ruled out as equilibria by the non-negativity constraint on aggregate real debt. In this section, we show that explosive paths for real assets are ruled out by the household transversality condition. Our proof strategy entails decomposing the household transversality condition and aggregating across households to show that the rate of growth of aggregate assets is bounded below by the discount rate ρ.

Consider a strictly positive sequence of real rates $(r_t)_{t\geq 0}$. The transversality condition in the stochastic economy is given by:

$$\lim_{t\to\infty} [\mathbb{E}_t \exp(-\rho t) u'(c_t(a_j, z_j)) a_j] = 0 \tag{E31}$$

The household Euler equation gives us a differential equation for the evolution of *expected* marginal utility.

$$\frac{\mathbb{E}_0[\mathrm{d}u'(c_t(a_j, z_j))]}{\mathbb{E}_0[u'(c_t(a_j, z_j))]} = (\rho - r_t)\mathrm{d}t \tag{E32}$$

We may solve this ordinary differential equation to obtain

$$\mathbb{E}_0[u'(c_t(a_j, z_j))] = u'(c_0(a_{j0}, z_{j0})) \exp\left(\rho t - \int_0^t r_s \mathrm{d}s\right) \tag{E33}$$

We next decompose the expectation term in the household transversality condition:

$$\begin{aligned} \lim_{t\to\infty} [\mathbb{E}_0 \exp(-\rho t) u'(c_t(a_j, z_j)) a_j] = \lim_{t\to\infty} [&\exp(-\rho t)\mathbb{E}_0\left[u'(c_t(a_j, z_j))\right]\mathbb{E}_0\left[a_j\right] \\ &+ \exp(-\rho t)\mathrm{Cov}_0(u'(c_t(a_j, z_j)), a_j)] \end{aligned} \tag{E34}$$

where the covariance is conditional on the households' time-zero information set. We may substitute for the first term using (E33) to obtain:

$$\begin{aligned} \lim_{t\to\infty} \Bigg[&\exp\left(-\int_0^t r_s \mathrm{d}s\right) u'(c_0(a_{j0}, z_{j0}))\mathbb{E}_0\left[a_j\right] \\ &+ \exp(-\rho t)\mathrm{Cov}_0(u'(c_t(a_j, z_j)), a_j)\Bigg] = 0 \end{aligned} \tag{E35}$$

We may also bound the covariance term via the Cauchy-Schwarz inequality to obtain

$$\begin{aligned} \exp(-\rho t)\mathrm{Cov}_0(u'(c_t(a_j, z_j)), a_j)| &\leq \exp(-\rho t)\sqrt{\mathrm{Var}_0(u'(c_t(a_j, z_j))}\sqrt{\mathrm{Var}_0(a_j)} \\ &\leq \exp(-\rho t)\frac{y_{min}^{-\gamma}}{2}\sqrt{\mathbb{E}_0[(a_j)^2]} \end{aligned}$$

where last the inequality has made use of the fact that $u'(c_{jt}) \leq y_{min}^{-\gamma}$and the Popoviciu bound on variances (Bhatia and Davis, 2000). Finally, we provide a bound on the variance of individual asset holdings. If asset holdings are uniformly bounded, the bound is trivially zero. So we only need to concern ourselves with cases in which individual assets may diverge to infinity. In these cases, we can use standard results on the asymptotic behaviour of the consumption function to provide an upper bound on assets (Benhabib et al., 2015; Achdou et al., 2022). In particular, we have:

$$\lim_{a_j \to \infty} \frac{\phi_t a_j}{c_{jt}} = 1 \tag{E36}$$

where $\phi_t > 0$. We may then use the household budget constraint to show that assets grow at a rate $r_t - \phi_t$ asymptotically, which yields the bound

$$a_j \leq \Xi \exp\left(\int_0^t (r_s - \phi_s)\mathrm{d}s\right), \quad \text{a.s.} \tag{E37}$$

for some finite $\Xi > 0$. Using the Popoviciu inequality once again, we obtain

$$|\exp(-\rho t)\mathrm{Cov}_0(u'(c_{jt}), a_j)| \leq \exp(-\rho t)\frac{y_{min}^{-\gamma}}{4}\Xi \exp\left(\int_0^t (r_s - \phi_s)\mathrm{d}s\right) \tag{E38}$$

Under the assumption that there exists some $t' > 0$ such that $r_t \leq \rho$ for $t \geq t'$, the right-hand side vanishes as we take $t \to \infty$. Section N.2 provides sufficient condition for $r_t < \rho$ for all $t \geq 0$.

We now show that (E35) precludes explosive paths for real aggregate debt. In particular, we show that

$$\lim_{t \to \infty}\left[\exp\left(\int_0^t -r_s \mathrm{d}s\right) a_t\right] = 0 \tag{E39}$$

where a_t is the amount of aggregate asset holdings in the economy at time t. To this end, we integrate (E35) over households to obtain:

$$\begin{aligned}
&\lim_{t \to \infty}\left[\int_{a,z} \mathbb{E}_0 \exp(-\rho t) u'(c_0(a,z)) a \mathrm{d}G_t(a,y)\right] \\
&\leq \lim_{t \to \infty} m \left[\exp\left(-\int_0^t r_s \mathrm{d}s\right) \underbrace{\int_{a,y} \mathbb{E}_{a_0=a}\left[a_t\right] \mathrm{d}G_t(a,z)}_{\bar{a}_t}\right] = 0
\end{aligned}$$

where m is an upper bound on marginal utility at $t = 0$: $u'(c_0^i(a^i)) \leq m \quad \forall a, y \in \text{supp } G_0(a, y)$, a.e. and where $G_t(\cdot, \cdot)$ is the distribution over assets and income at time t. Note that term in the integral in the second inequality is equal to aggregate asset holdings by the exact law of large numbers (Duffie and Sun, 2012). This shows that no equilibria exist in which government debt explodes upwards.

F Extended Model for Quantitative Analysis

F.1 Model With Borrowing

In this section, we describe how the model is consistent with a negative lower bound on real household assets and costly borrowing. Households face a borrowing limit (a negative scalar) expressed in real terms:

$$\frac{A_{jt}}{P_t} \geq \underline{\tilde{a}}_t \tag{F1}$$

In order for the borrowing constraint to be consistent with balanced growth, we assume that $\underline{\tilde{a}}_t$ grows at the rate of real output, $\underline{\tilde{a}}_t = y_0 e^{gt} \underline{a}$ for some $\underline{a} < 0$. Note that this implies that $a_{jt} \geq \underline{a}$. Furthermore, we assume that borrowing is costly. Households face a wedge $\vartheta \geq 0$ on the real interest rate when borrowing, so that the interest rate they pay on debt is $\vartheta + r_t$. This borrowing wedge creates deadweight loss in output that is equal to the wedge multiplied by the amount of assets borrowed. This implies that aggregate consumption is slightly less than output (a difference around 0.3% of steady-state output in our calibration). The associated boundary condition for the HJB equation (8) is:

$$\partial_a v_t(0, z) \geq (z - \tau_t(z) + (r_t + \vartheta)\underline{a})^{-\gamma} \tag{F2}$$

The government accumulation equation continues to follow equation (18), with the understanding that $b_t \geq 0$, so that the government can lend, but not borrow.

F.2 Model With Long-Term Debt

The government now issues two securities: short-term debt B_t^s that pays a nominal rate i_t, and long-term debt B_t^l. Long-term debt takes the form of depreciating consoles that depreciate at a rate $\delta > 0$, and that yield a flow coupon payment of $\chi > 0$ as in Cochrane (2001). We let q_t denote the market value of this long-term bond. The government's budget constraint can be written as:

$$\mathrm{d}B_t^s + q_t \mathrm{d}B_t^l = [iB_t^s + (\chi - \delta q_t)B_t^l - P_t s_t]\mathrm{d}t \tag{F3}$$

The intuition for this equation is as follows. The right-hand side is the government's nominal deficit that consists of the primary deficit $-P_t s_t$, interest payments on short-term debt iB_t^s, and coupon payments plus redemption of long-term debt $(\chi - \delta q_t)B_t^l$. Whenever the deficit is greater than zero, the government must issue additional debt. It can do so either by issuing additional short-term debt or by issuing additional long-term debt at the price of q_t.

Similarly, we may define the nominal short- and long-term debt holdings of household j at time t as A_{jt}^s and A_{jt}^l, respectively. The household budget constraint becomes

$$\mathrm{d}A_{jt}^s + q_t \mathrm{d}A_{jt}^l = [i_t A_{jt}^s + (\chi - \delta q_t)A_{jt}^l + (z_{jt} - \tau_t(z_{jt}))P_t y_t - P_t \tilde{c}_{jt}]\mathrm{d}t \tag{F4}$$

We define the market value of total government debt outstanding as $B_t := B_t^s + q_t B_t^l$ and the total value of household assets as $A_{jt} := A_{jt}^s + q_t A_{jt}^l$. We also define de-trended real debt and assets relative to GDP as in the main text:

$$b_t = \frac{B_t}{P_t y_0 e^{gt}} \quad \text{and} \quad a_{jt} = \frac{A_{jt}}{P_t y_0 e^{gt}} \tag{F5}$$

The next proposition demonstrates that the maturity structure is irrelevant for government debt: b_t and a_{jt} are the only state variables in this economy, except at impact after an unanticipated shock when the portfolio share held in short vs long-term bonds matters to determine capital gains/losses from the jump in q_t.

Proposition 3. *The household budget constraint follows (6) and the real government budget constraint follows (18) for $t > 0$. Moreover, the price of long-term debt satisfies the following differential equation for $t > 0$:*

$$\frac{\dot{q}_t}{q_t} + \frac{\chi - \delta q_t}{q_t} = i_t \tag{F6}$$

Proof. See Not For Publication Appendix N.3 □

Table G1: Maximal Deficits under Varying Calibration Targets

Calibrated ρ (Target s^*)	Maximal Deficit			
	$\tau_0 \uparrow$	$\tau_1 \uparrow$	$\tau_0 \uparrow, \underline{a} = 0$	$\tau_0 \uparrow, \tau_1 = 0$
5.6% p.a. (−2.0%)	4.17%	4.61%	5.80%	9.30%
4.72% p.a. (−2.5%)	4.38%	4.87%	5.96%	9.59%
4.08% p.a. (−3.0%)	4.60%	5.16%	6.12%	9.90%
3.48% p.a. (−3.5%)	4.84%	5.46%	6.28%	10.21%
$\rho \approx 0$	6.54%	7.79%	7.48%	12.24%

Note: This table depicts maximal deficits under different calibration targets. Each row calibrates the discount rate ρ to match the corresponding steady-state deficit target s^*, holding all other parameters fixed as in Table 1. The last row depicts a "robust" upper bound for deficits by setting $\rho = 1e-7$. The columns report how the maximal deficit depends on the mode of fiscal adjustment: Column 2 increases lump-sum transfers; Column 3 reduces proportional taxes; Column 4 increases lump-sum transfers and sets the borrowing constraint to zero; Column 5 increases lump-sum transfers and sets proportional taxes to zero.

This proof shows that an economy with long-term debt collapses into an economy with short-term debt in the absence of uncertainty. Equation (F6) is an arbitrage relationship between short- and long-term debt. In equilibrium, households are indifferent between the two assets. Hence, long-term debt will only matter for inflation dynamics insofar there is an unanticipated change in nominal rates i_t. Equation (F6) is a forward-looking equation.

G Robustness of Maximum Deficits

Table G1 contains the results of the robustness analysis on maximum sustainable deficits. We calibrate ρ to match different steady-state deficits given a steady-state debt-to-GDP ratio of 1.03. In all examples, the steady-state deficit is increased via higher lump-sum taxation. Increasing the steady-state deficit target lowers the calibrated discount rate and induces a rightwards shift in the household steady-state asset demand curve. Thus, households are incentivized to hold the same aggregate quantity of debt despite the lower steady-state real rate.

Next, we explore the sensitivity of our quantitative results to the household discount rate. Columns 2-5 of Table G1 depict the maximal deficit that can be implemented from higher lump-sum transfers, from lower proportional taxes, from raising lump-sum transfers under the assumption that households cannot borrow, and from raising lump-sum transfers when proportional taxes are set to zero. As explained in the main text, a key qualitative finding is that maximal deficits are higher when the degree of insurance in the economy is lower. We also report the maximal deficits under different financing configurations when the household discount rate is very close to zero. This is meant to capture an upper bound for the maximal deficit when the discount rate cannot be directly observed by the economic analyst.

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Not For Publication Appendix

N.1 Derivations for the Real Rate Functional

Recall equation (E20):

$$\frac{\mathcal{C}_t^{\mathrm{u}}}{\gamma}(r_t-\rho)+\mathcal{C}_t^u\tilde{\mathbb{E}}_t^{\mathrm{u}}\left[\frac{1}{\gamma}\sum_{z'}\lambda_{z_jz'}\left(\frac{c_t(a_j,z')}{c_{jt}}\right)^{-\gamma}+\sum_{z'}\lambda_{z_jz'}\frac{c_t(a_j,z')}{c_{jt}}\right]+\mathcal{C}_t^{\mathrm{c}}\tilde{\mathbb{E}}_t^{\mathrm{c}}\left[\sum_{z'}\lambda_{z_jz'}\left(\frac{c_t(a_j,z')}{c_{jt}}-1\right)\right]=0 \tag{N1}$$

Consider the second term of this expression, and take a second-order approximation of $c_t\left(a_j,z'\right)^{-\gamma}$ around $c_t\left(a_j,z_j\right)$:

$$\begin{aligned} c_t\left(a_j,z'\right)^{-\gamma} &= c_t\left(a_j,z_j\right)^{-\gamma}-\gamma c_t\left(a_j,z_j\right)^{-\gamma-1}\left[c_t\left(a_j,z'\right)-c_t\left(a_j,z_j\right)\right] \\ &\quad +\frac{1}{2}\gamma\left(\gamma+1\right)c_t\left(a_j,z_j\right)^{-\gamma-2}\left[c_t\left(a_j,z'\right)-c_t\left(a_j,z_j\right)\right]^2 \end{aligned}$$

Divide by both sides by $c_t\left(a_j,z_j\right)^{-\gamma}$ and obtain

$$\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}\right)^{-\gamma}=1-\gamma\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}-1\right)+\frac{1}{2}\gamma\left(\gamma+1\right)\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}-1\right)^2$$

Compute the conditional mean with respect to z'

$$\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}\right)^{-\gamma} = -\gamma\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}\right)+\frac{1}{2}\gamma\left(\gamma+1\right)\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}-1\right)^2$$

where we used the property that $\sum_{z'}\lambda_{z',z_j}=0$.

Dividing by γ and rearranging, we obtain

$$\frac{1}{\gamma}\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}\right)^{-\gamma}+\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}\right)=\left(\frac{\gamma+1}{2}\right)\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}-1\right)^2$$

Substituting this expression into (E20), we obtain the expression for the real rate functional in equation (E21) and in the main text:

$$\frac{\mathcal{C}_t^{\mathrm{u}}}{\gamma}(r_t-\rho)+\frac{\gamma+1}{2}\mathcal{C}_t^u\tilde{\mathbb{E}}_t^{\mathrm{u}}\left[\sum_{z'}\lambda_{z',z_j}\left(\frac{c_t\left(a_j,z'\right)}{c_t\left(a_j,z_j\right)}-1\right)^2\right]+\mathcal{C}_t^{\mathrm{c}}\tilde{\mathbb{E}}_t^{\mathrm{c}}\left[\sum_{z'}\lambda_{z_jz'}\left(\frac{c_t(a_j,z')}{c_{jt}}-1\right)\right]=0.$$

N.2 Household Problem with Diffusion Process

This section sets up an economy in which income follows a diffusion process. We derive as an auxiliary result that $r_t < \rho$ for all $t \geq 0$ in this economy.

Concretely, we assume that household income follows a diffusion process given by

$$\mathrm{d}z_{jt} = \mu_z(z_{jt})\mathrm{d}t + \sigma_z(z_{jt})\mathrm{d}B_{jt} \tag{N2}$$

where B_{jt} is adapted Brownian motion, independent across j, and $\mu_z(\cdot) : \mathbb{R} \to \mathbb{R}$ and $\sigma_z(\cdot) : \mathbb{R} \to \mathbb{R}^+$ are twice-differentiable functions.[26] We further assume that (N2) admits a stationary distribution. The household problem now satisfies the following HJB equation:

$$\begin{aligned} \rho v_t(a,z) - \partial_t v_t(a,z) = \max_c \frac{c^{1-\gamma}}{1-\gamma} &+ \partial_a v_t(a,z)\left[r_t a + z - \tau_t(z) - c\right] \\ &+ \mu_z \partial_z v_t(a,z) + \frac{1}{2}\sigma_z^2 \partial_{zz}^2 v_t(a,z) \end{aligned} \tag{N3}$$

together with the boundary condition $\partial_a v_t(0,z) \geq (z - \tau_t(z))^{-\gamma}$. A solution to the HJB equation alongside (11) solves the household problem. The associated KFE equation is:

$$\partial_t g_t(a,z) = -\partial_a[g_t(a,z)\varsigma_t(a,z)] - \partial_z[\mu_z(z)g_t(a,z)] + \frac{1}{2}\partial_{zz}^2[\sigma_z^2(z)g_t(a,z)] \tag{N4}$$

Expected Consumption Dynamics. We now derive the expected consumption dynamics for unconstrained households. Following exactly the same steps outlined in Appendix E.3 for the case in which income follows a Poisson process, we can derive an Euler equation for unconstrained households:

$$\begin{aligned} (\rho - r_t)u'(c_t(a,z)) = {} & \mu_z(z)u''(c_t(a,z))\partial_z c_t(a,z) \\ & + \frac{1}{2}\sigma_z^2(z)\left(u''(c_t(a,z))\partial_{zz}^2 c_t(a,z) + u'''(c_t(a,z))(\partial_z c_t(a,z))^2\right) \\ & + u''(c_t(a,z))[\partial_t c_t(a,z) + \varsigma_t(a,z)\partial_a c_t(a,z)] \end{aligned} \tag{N5}$$

[26] Note that z_{jt} never becomes negative under an appropriate boundary condition for $\sigma_z(z)$ as $z \to 0$.

We can also use Ito's lemma on $c_t(a_{jt}, z_{jt})$ to obtain

$$\begin{aligned} \mathrm{d}c_t(a_{jt}, z_{jt}) =& [\partial_t c_t(a_{jt}, z_{jt}) + \varsigma_t(a_{jt}, z_{jt})\partial_a c_t(a_{jt}, z_{jt})]\mathrm{d}t \\ &+ [\mu_z(z_{jt})\partial_z c_t(a_{jt}, z_{jt}) + \frac{1}{2}\sigma_z^2(z_{jt})\partial_{zz}^2 c_t(a_{jt}, z_{jt})]\mathrm{d}t \\ &+ \sigma_z(z_{jt})\partial_z c_t(a_{jt}, z_{jt})\mathrm{d}B_{jt} \end{aligned} \tag{N6}$$

Taking expectations of the above equation, combining it with (N5), and imposing that u is CRRA with coefficient of risk aversion γ yields the expected consumption dynamics for unconstrained households:

$$\frac{\mathbb{E}_t[\mathrm{d}c_{jt}]}{c_{jt}\mathrm{d}t} = \frac{1}{\gamma}(r_t - \rho) + \frac{\gamma+1}{2}\sigma_z^2(z_{jt})\left(\frac{\partial_z c_t(a_{jt}, z_{jt})}{c_t(a_{jt}, z_{jt})}\right)^2 \tag{N7}$$

The first term on the right-hand side of (N7) represents the intertemporal motive and the second one is the precautionary saving motive ($\gamma + 1$ is the coefficient of relative prudence).

Constrained households simply consume their (after-tax) income. Hence, their consumption dynamics are

$$\mathrm{d}c_{jt} = [\tilde{\mu}_z(z_{jt})]\mathrm{d}t + \tilde{\sigma}_z(z_{jt})\mathrm{d}B_{jt} \tag{N8}$$

where

$$\tilde{\mu}_z(z) = \mu_z(z)(1 - \partial_z\tau(z)) + \frac{1}{2}\sigma_z^2(z)\partial_{zz}\tau_t(z) - \partial_t\tau_t(z) \tag{N9}$$

and

$$\tilde{\sigma}_z(z) = \sigma_z(z)(1 - \partial_z\tau_z(z)) \tag{N10}$$

The expected consumption dynamics of constrained households are therefore given by

$$\frac{\mathbb{E}_t[\mathrm{d}c_{jt}]}{\mathrm{d}t} = \tilde{\mu}_z(z_{jt}) \tag{N11}$$

Derivation of Interest Rate Functional. Integrating over the consumption dynamics of unconstrained households and making use of the fact that

$$\int_j \frac{\mathrm{d}c_{jt}}{\mathrm{d}t}\mathrm{d}j = 0$$

yields

$$
\begin{aligned}
0 = &\int_{j:u} \frac{1}{\gamma}(r_t - \rho)c_{jt}\mathrm{d}j + \int_{j:u} \frac{\gamma+1}{2} c_t(a_{jt}, z_{jt}) \left(\frac{\sigma_z(z_{jt})\partial_z c_t(a_{jt}, z_{jt})}{c_t(a_{jt}, z_{jt})} \right)^2 \mathrm{d}j \\
&+ \int_{j:c} \tilde{\mu}_z(z_{jt})\mathrm{d}j
\end{aligned}
\tag{N12}
$$

where we have used (N7) and (N11). Rearranging, we obtain:

$$
r_t = \rho - \frac{\frac{\gamma(\gamma+1)}{2}\int_{j:u} c_t(a_{jt}, z_{jt}) \left(\frac{\sigma_z(z_{jt})\partial_z c_t(a_{jt}, z_{jt})}{c_t(a_{jt}, z_{jt})} \right)^2 \mathrm{d}j + \gamma \int_{j:c} \tilde{\mu}_z(z_{jt})\mathrm{d}j}{\int_{j:u} c_t(a_{jt}, z_{jt})\mathrm{d}j} \tag{N13}
$$

Note that this implies that $r_t < \rho$ for all $t \geq 0$ (not just in steady-state) if no households are constrained, or if $\int_{j:c} \tilde{\mu}_z(z_{jt})\mathrm{d}j > 0$, so that constrained households expect their (post-tax) earnings to increase, on average.

We may also write the formula analogously as the one in the main text for the Poisson income process (28):

$$
0 = \frac{\mathcal{C}_t^{\mathrm{u}}}{\gamma}(r_t - \rho) + \frac{\gamma+1}{2}\mathcal{C}_t^{\mathrm{u}}\tilde{\mathbb{E}}_t^{\mathrm{u}}\left[\sigma_z^2(z_j)\left(\frac{\partial_z c_t(a_j, z_j)}{c_t(a_j, z_j)}\right)^2\right] + \mathcal{C}_t^{\mathrm{c}}\tilde{\mathbb{E}}_t^{\mathrm{c}}\left[\frac{\tilde{\mu}_z(z_j)}{c_t(a_j, z_j)}\right] \tag{N14}
$$

where $\mathcal{C}_t^{\mathrm{u}} = \int_{j:u} c_t(a_j, z_j)\mathrm{d}j$, $\mathcal{C}_t^{\mathrm{c}} = \int_{j:c} c_t(a_j, z_j)\mathrm{d}j$, and where $\tilde{\mathbb{E}}_t^{\mathrm{u}}$ and $\tilde{\mathbb{E}}_t^{\mathrm{c}}$ are the consumption-weighted means across all unconstrained and constrained households, respectively.

As for the Poisson income process case, the first term on the right-hand side of (N14) captures the intertemporal substitution motive, the second term captures the precautionary saving motive related to prudence ($\gamma + 1$ is the coefficient of relative prudence with CRRA utility), and the third term captures the precautionary saving motive related to the severity of borrowing constraints: the more stringent are constraints at t, the larger will be the share of consumption of constrained households $\mathcal{C}_t^{\mathrm{c}}$ and the higher will be their expected consumption growth $\tilde{\mathbb{E}}_t^{\mathrm{c}}[\tilde{\mu}_z(z_j)/c_t(a_j, z_j)]$.

N.3 Proof for the Model With Long-Term Debt

Proposition 4. *The household budget constraint follows* (6) *and the real government budget constraint follows* (18) *for* $t > 0$. *Moreover, the price of long-term debt satisfies* *the following differential equation for $t > 0$:*

$$\frac{\dot{q}_t}{q_t} + \frac{\chi - \delta q_t}{q_t} = i_t \tag{N15}$$

Proof. We define households' long-term debt savings via the control variable $u := \mathrm{d}A^l$. Using Equation F4, we can write the households' HJB equation as:

$$\tilde{\rho} v_t(A^l, A^s, z) - \partial_t v(A^l, A^s, z) =$$
$$\max_{c.u} \frac{c^{1-\gamma}}{1-\gamma} + \tilde{s}_t \partial_{A^s} v_t(A^l, A^s, z) + \partial_{A^l} v_t(A^l, A^s, z) u + \sum_{z' \neq z} \lambda_{zz'} [v_t(A^l, A^s, z') - v_t(A^l, A^s, z)]$$

where

$$\tilde{s}_t := i_t A^s + (\chi - \delta q_t) A^l + (z - \tau(z)) P_t y_t - P_t \tilde{c}_t - q_t u$$

The first-order condition with respect to u is given by:

$$q_t \partial_{A^s} v(A^l, A^s, z) = \partial_{A^l} v_t(A^l, A^s, z) \tag{N16}$$

We may differentiate with respect to time to obtain:

$$q_t \partial^2_{A^s,t} v_t(A^l, A^s, z) + \partial_t q_t \partial_{A^s} v(A^l, A^s, z) = \partial_{A^l,t} v_t(A^l, A^s, z) \tag{N17}$$

The envelope condition for the HJB with respect to A^l is:

$$\tilde{\rho} \partial_{A^l} v_t - \partial^2_{t,A^l} v_t = \tilde{s}_t \partial^2_{A^s,A^l} v_t + (\chi - \delta q_t) \partial_{A^l} v_t + u \partial^2_{A^l} v_t + \sum_{z' \neq z} \lambda_{zz'} [\partial_{A^l} v_t - \partial_{A^l} v_t] \tag{N18}$$

Similarly, the envelope condition for the HJB with respect to A^s is:

$$\tilde{\rho} \partial_{A^s} v_t - \partial^2_{t,A^s} v_t = \tilde{s}_t \partial^2_{A^s} v_t + i_t \partial_{A^s} v_t + u \partial^2_{A^l,A^s} v_t + \sum_{z' \neq z} \lambda_{zz'} [\partial_{A^s} v_t - \partial_{A^s} v_t] \tag{N19}$$

Multiplying (N19) by q_t, subtracting Equation (N18) from (N19) and using (N16) and (N17) yields:

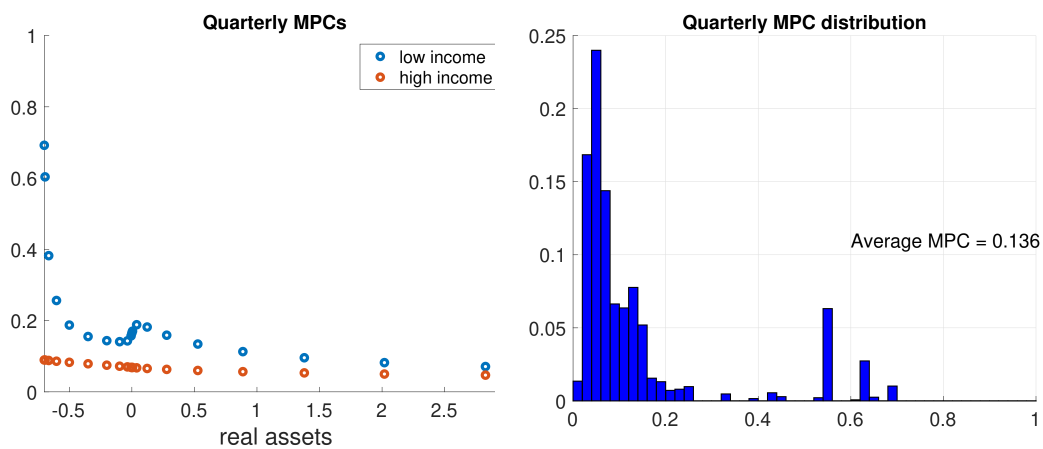
$$(q_t i_t - (\chi - \delta q_t) - \partial_t q_t) \partial_{A^l} v_t = 0 \tag{N20}$$

By market clearing, we must have $\partial_{A^l} v_t > 0$ (otherwise no long-term debt would be purchased in equilibrium). Hence, we have the arbitrage relationship:

$$\frac{\dot{q}_t}{q_t} + \frac{\chi - \delta q_t}{q_t} = i_t \tag{N21}$$

Differentiating $B_t = q_t B_t^l + B_t^s$ with respect to time and using (F3) then yields Equation (14), which can be written in real terms. This completes the proof. □

N.4 Supplement on Wealth Distribution and MPCs



(a) MPCs by income

(b) Distribution of MPCs

Figure N1: MPCs in the calibrated steady-state

This section provides some additional detail on the MPCs in the calibrated steady-state. Figure N1a shows the dependence of marginal propensities to consume on real assets, disaggregated by the highest and lowest income draws. The plotted MPCs are the quarterly marginal propensities to consume from an unanticipated \$500 income gain.

MPCs are not monotonically decreasing in real assets because there is a borrowing wedge. Households with zero assets therefore have a high marginal propensity to consume because of the discontinuous cost of borrowing (Kaplan and Violante, 2014). Note that the MPCs of high income households lie uniformly below the MPCs of low income households.

Figure N1b plots the distribution of MPCs in the calibrated steady-state. A large number of households have an MPC of around 0.20 and hold zero assets. The average quarterly MPC in the economy is around 0.14, which is in line with commonly estimated values for marginal propensities to consume (Jappelli and Pistaferri, 2010).

N.5 Inflationary Effects of Pure Redistribution

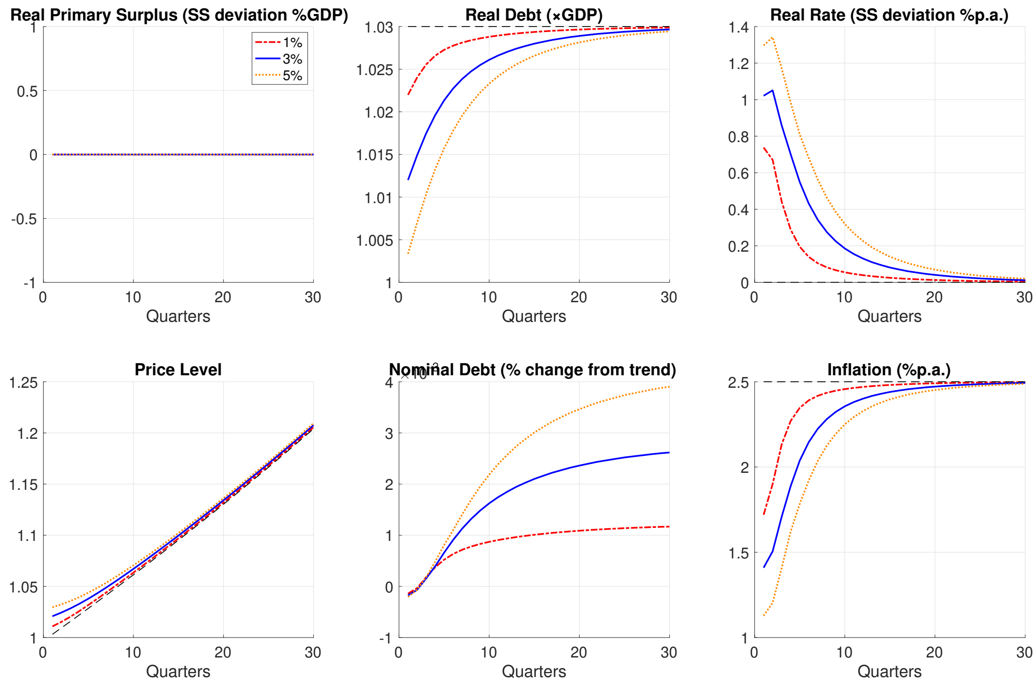
A comparison of the heterogeneous agent and representative agent economies in the preceding experiments suggests that redistribution itself has effects on the price level and inflation that are independent of the overall level of surpluses and nominal government debt. To emphasize the inflationary effects of redistribution, Figure N2 shows simulations from purely redistributive shocks. We consider one-time wealth taxes levied on the top 10% of the wealth distribution, the proceeds of which are redistributed lump-sum to the bottom 60%. Although these shocks do not entail any new issuance of government debt or any change in primary deficits, they do cause a period of inflation. The redistribution causes upward pressure on consumption because low-wealth households have higher average MPCs than high wealth households. Equilibrium is achieved through a period of higher real interest rates. The corresponding lower government revenues require a downward revaluation in real debt through a jump in the price level.

Inflationary Effects of Proportional Wealth Taxes. We contrast this experiment with another version of wealth taxation. Consider an economy where the government levies a proportional wealth tax at a rate of τ_b so that total primary surpluses are $s^* + \tau_b b_t$ (where s^* are surpluses net of revenue from the wealth tax). The real government budget constraint becomes:

$$\mathrm{d}b_t = \left[(r_t - \tau_b)b_t - s^*\right]\mathrm{d}t. \tag{N22}$$

The wealth tax appears in the household budget constraint in a similar fashion, as it increases the after-tax real rate paid to the government, $r_t - \tau_b$. Changes in τ_b therefore only affect the inflation rate through the Fisher equation, but otherwise leave the real economy and the initial price level unchanged.

Figure N2



Note: Impulse responses to a temporary increase in the wealth tax, with the proceedings distributed lump-sum, for various values of the wealth tax. In all experiments, the wealth tax is levied on the top 10% of the wealth distribution, the proceeds of which are redistributed lump-sum to the bottom 60%. The dashed black line plots the pre-shock trend.

N.6 Endogenous Output

In this subsection, we outline an economy in which labor is a variable input in production. Next, we discuss how endogenous output affects price level and inflation dynamics in response to unanticipated shocks.

N.6.1 Set-Up

Households. The set-up of the household problem closely follows that of the main text. However, we assume that households choose real consumption flows $\tilde{c}_{jt}$ and hours worked ℓ_{jt} to maximize

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\frac{\tilde{c}_{jt}^{1-\gamma}}{1-\gamma} - \phi_t^{1-\gamma} \frac{\ell_{jt}^{1+\psi}}{1+\psi} \right] \mathrm{d}t \tag{N23}$$

where the expectation is taken with respect to households' efficiency units of labour z_{jt}. The exponent $\psi > 0$ is the inverse of the Frisch elasticity of labor supply. The term ϕ_t is a time-varying constant that augments the labor disutility in order to allow the economy to be consistent with balanced growth when $\gamma \neq 1$. Concretely, we assume that

$$\phi_t = \tilde{\phi} e^{gt} \tag{N24}$$

where $\tilde{\phi} > 0$ and $g > 0$ is the growth rate of the economy. This formulation implies that a stationary equilibrium exists. Moreover, the distribution of hours across households is constant in the stationary equilibrium.[27] The households nominal budget constraint therefore satisfies

$$\mathrm{d}A_{jt} = [i_t A_{jt} + (1 - \tau_{1t}) z_{jt} P_t w_t \ell_{jt} - P_t \tilde{c}_{jt} + P_t \tau_{0t}]\mathrm{d}t \tag{N25}$$

where w_t is the real wage rate for effective labor services at time t, τ_{0t} is a lump-sum payment and τ_{1t} is a constant proportional tax rate. We assume that τ_{0t} grows at a rate $g > 0$ in order to ensure that a stationary equilibrium exists:

$$\tau_{0t} = \tilde{\tau}_0 e^{gt} \tag{N26}$$

Finally, the stochastic process for z_{jt} and the definition of de-trended real variables for the evolution of real debt are identical to those of the main text.

Firms. We assume that perfectly competitive firms hire labor to produce output y_t with the constant returns to scale (CRS) production function

$$y_t = \Theta_t L_t \tag{N27}$$

where Θ_t is aggregate total factor productivity that grows at a rate $g > 0$ and L_t are total effective hours:

$$L_t := \int_{j \in [0,1]} z_{jt} \ell_{jt} \mathrm{d}j \tag{N28}$$

CRS implies that the real wage rate w_t is equal to Θ_t for all $t \geq 0$.

[27] We intentionally assume separability between hours and consumption in the instantaneous utility function so as to maximize comparability between the economy with endogenous output presented in this subsection and the endowment economy presented in the main text. In particular, the endowment economy can be closely approximated for large ψ and a given calibrated $\tilde{\phi}$. We note, however, that preferences by King et al. (1988) leave the key mechanisms unaffected.

Government. The dynamics for government debt are given by

$$\mathrm{d}B_t = [i_t B_t - s_t P_t y_t]\mathrm{d}t \tag{N29}$$

where s_t is the ratio of primary surpluses to output and is determined by the tax/transfer system and government consumption as

$$s_t = \frac{\tau_{0t}}{y_t} + \tau_{1t} - x_t \tag{N30}$$

where we have used the fact that, with competitive input and output markets, $y_t = w_t L_t$. De-trended real government debt then follows

$$\mathrm{d}b_t = [r_t b_t - s_t]\mathrm{d}t \tag{N31}$$

We do not consider unanticipated changes in the nominal rate in this section. Consequently, we assume an interest rate peg $i_t = i^*$ without loss of generality in analyzing real dynamics.

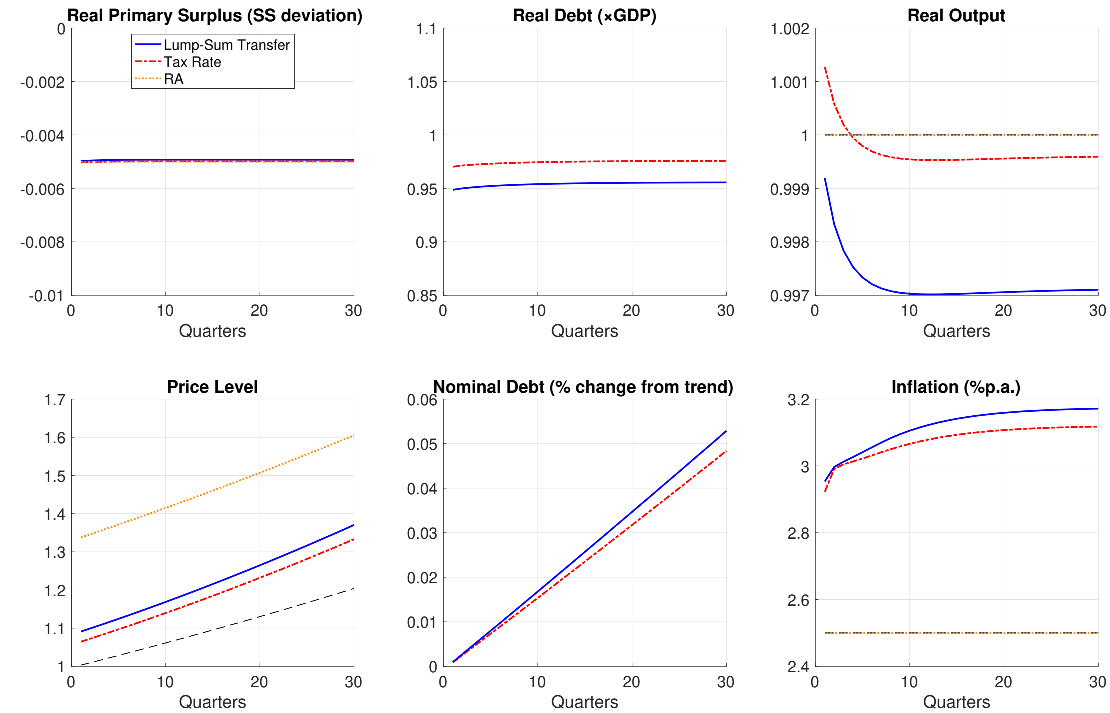
Calibration. Our calibration sets $\psi = 2$, so that the intensive-margin Frisch elasticity of labor supply is equal to one-half, in line with the recommendation of Chetty et al. (2011). Moreover, we calibrate $\tilde{\phi}$ so as to set total hours worked equal to unity. Allowing labor to adjust on the intensive margin provides additional insurance to households. As such, the discount rate increases to 6.1% p.a. (relative to 2.8% p.a. from the calibration in the main text) in order to match a debt-to-annual GDP ratio of 1.10. The values for the remaining parameters remain unchanged from Table 1.

N.6.2 Quantitative Exercise

We consider the economy's response to an increase in deficits. First, we consider the economy's response to a permanent change in $\tilde{\tau}_{0t}$ from 0.20 to 0.205, keeping τ_1 fixed. Second, we consider a permanent change in τ_{1t} from 0.320 to 0.325, keeping τ_{0t} fixed. These changes amount to a change in deficits from 2.0% to 2.5% of GDP, if output was unchanged (in line with the analysis of Section 5.4).

An increase in deficits due to a tax cut results in a smaller jump in the initial price level, relative to the transfer expansion case. The main reason is that lower taxation increases the labor supply (whereas a transfer expansion lowers it). The

Figure N3



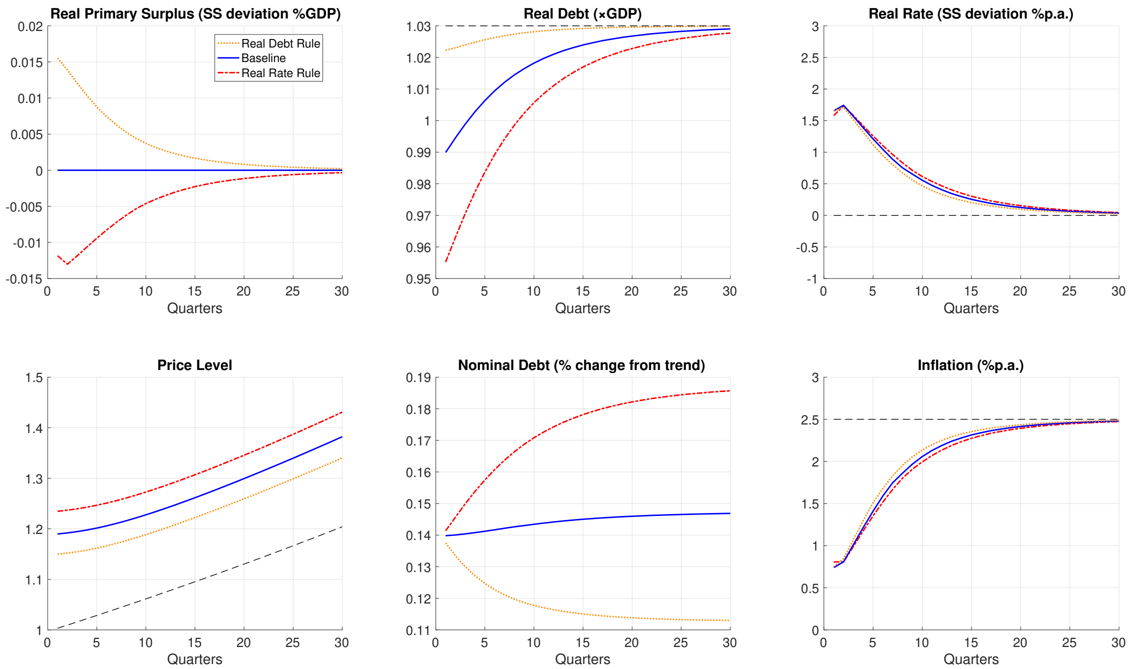
Note: Impulse responses to a permanent expansion in primary deficits in the economy with endogenous output. The dotted orange line shows the effects of a permanent reduction in surpluses in the Representative Agent model due to a change in transfers. The solid blue line labeled "Lump-Sum Transfer" illustrates the dynamics following an expansion of lump sum transfers. The dashed red line labeled "Tax Rate" illustrates the dynamics following a tax cut. The dashed black line plots the pre-shock trend. In all experiments, deficits increase by 0.5% of pre-shock GDP.

corresponding rise in output raises tax revenues and attenuates the long-run increase in primary deficits relative to the transfer expansion.[28]

In both economies, however, real output eventually *declines* relative to the representative agent benchmark. In order to understand this result, consider the tax cut experiment. There are two forces that contribute to an increase in labor supply. First, the tax cut directly raises the return to working, as explained above. Sec-

[28]The tax cut also increases precautionary motives by amplifying the volatility of post-tax earnings, in line with the reasoning of Section 5.4. The real interest rate therefore decreases relatively less. Since the government now finances its debt at a higher cost, this a force that contributes to a larger initial jump in the price level. However, this mechanism is dominated by the labor-supply channel.

Figure N4



Note: Impulse responses to targeted fiscal helicopter drop under alternative fiscal rules. The dotted orange line corresponds to the "real debt rule" of equation equations (D11) and the dashed red line corresponds to the "real rate rule" in equation (D1) with parameter values of $\phi_b = -0.5$ and $\phi_r = -3$, respectively. The dashed black line plots the pre-shock trend.

ond, households in the new steady-state hold lower amounts of wealth, on average. This gives rise to positive wealth effects which increase total hours worked. However, the new steady-state features a lower long-run real rate—a force only present in the heterogeneous agent economy. The reduction in the real rate increases consumption state-by-state due to the intertemporal savings motive, thereby reducing total hours worked. This last force is sufficiently strong that it counteracts the positive effect on output due to the lower tax rate and the change in the wealth distribution. Consequently, in the long run, output falls and deficits rise relative to the representative agent economy.

N.7 Helicopter Drop Under Other Surplus Rules

To justify focusing attention on the saddle-path equilibrium we are implicitly appealing to long-run inflation anchoring. As discussed in Section 3.2, surplus reaction rules are an alternative route to uniqueness. Figure N4 shows that the price level, real rate and inflation dynamics from the fiscal helicopter drop are not sensitive to using either of the two classes of surplus reaction rules in equations (D1) and (D11) that guarantee a unique equilibrium. However, the two rules differ in the direction that primary deficits respond to the fiscal helicopter drop. Under the real debt reaction rule, the downward revaluation of real debt from the initial burst of inflation leads the fiscal authority to cut deficits following the helicopter drop. Under the real rate reaction rule, the higher real interest rate leads to a temporary increase in deficits.[29]

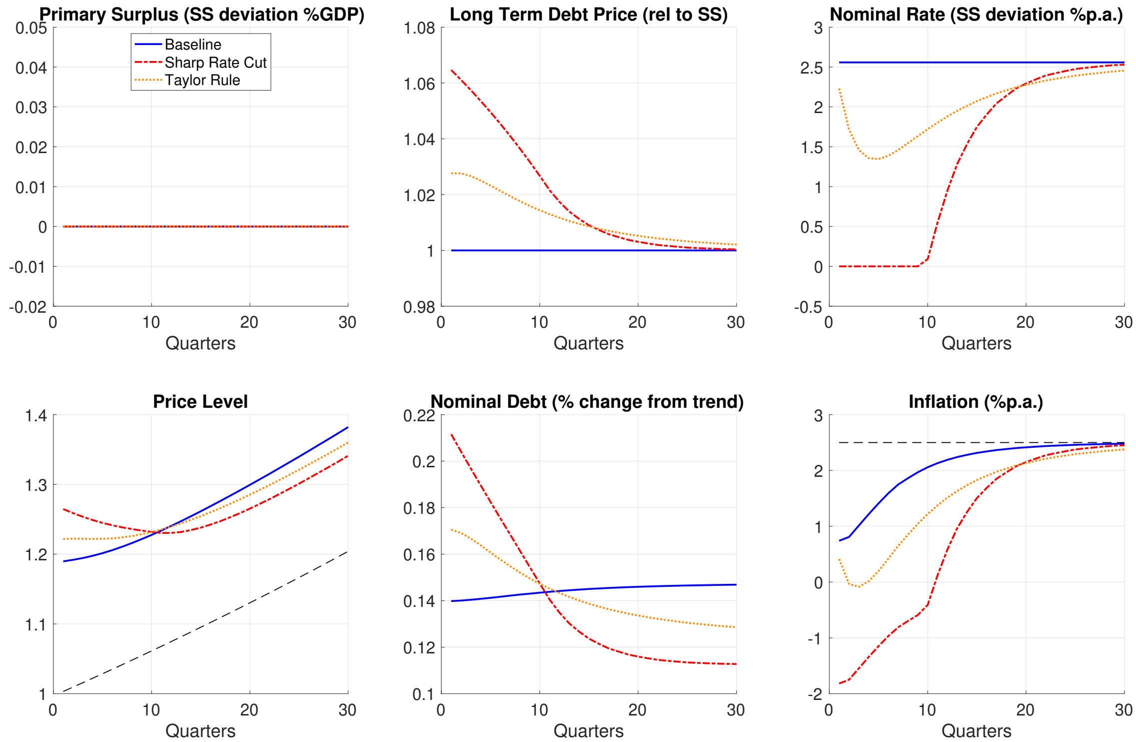
N.8 Helicopter Drop Under Different Monetary Responses

Throughout our previous simulations we have assumed that the central bank holds the nominal rate constant at 1.5% in response to the helicopter drop. Figure N5 reports results from two alternative experiments in which nominal rates are lowered at the same time as the fiscal expansion, like was done by central banks around the world in 2020. The dotted orange line labeled "Taylor rule" shows the effects of following a lagged Taylor rule as in equation (21), with a feedback parameter $\theta_m = 1$ and a coefficient on inflation $\phi_m = 0.5$. The dashed red line labeled "sharp rate cut" shows the implication of an immediate cut in the short-term interest rate all the way to zero, followed by a gradual normalization after 9 quarters. For comparison, the blue line labeled "baseline" reproduces the dynamics holding the nominal rate constant.

Monetary policy is an important driver of nominal aggregates. The behavior of long-term government bond prices is central to these dynamics. As explained in Sims (2011) and Cochrane (2018), a lower short-term nominal rate leads to a higher price of long-term government bonds through the yield curve. Thus, the overall price level must rise by a larger amount to achieve the same-size drop in the real value of

[29] Cochrane (2023) argues that following an expansion in nominal debt, a reduction in primary deficits is more in line with the historical record for the U.S. However, Jacobson et al. (2023) discuss an important historical example in which new debt was issued with the explicit intention of generating inflation by committing to not raise future surpluses to repay the debt.

Figure N5



Note: Impulse response to targeted fiscal helicopter drop under different monetary policy responses. The dotted orange line corresponds to the Taylor rule in equation (21) with $\theta_m = 1$ and $\phi_m = 0.5$. The dashed red line is a temporary cut of nominal rates all the way to the zero lower bound. The dashed black line plots the pre-shock trend.

outstanding government debt. Figure N5 shows that looser monetary policy causes an additional 4 to 7 percentage point increase in the price level upon impact, relative to the baseline with a nominal rate peg. The strength of this force is determined by the average duration of debt: the longer the duration, the bigger the initial jump in the price level.

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